ICRAR

International Centre for Radio Astronomy Research

Beyond Light Curve Smoothing: Gaussian Processes as Vivid Descriptors of Variables and Transients

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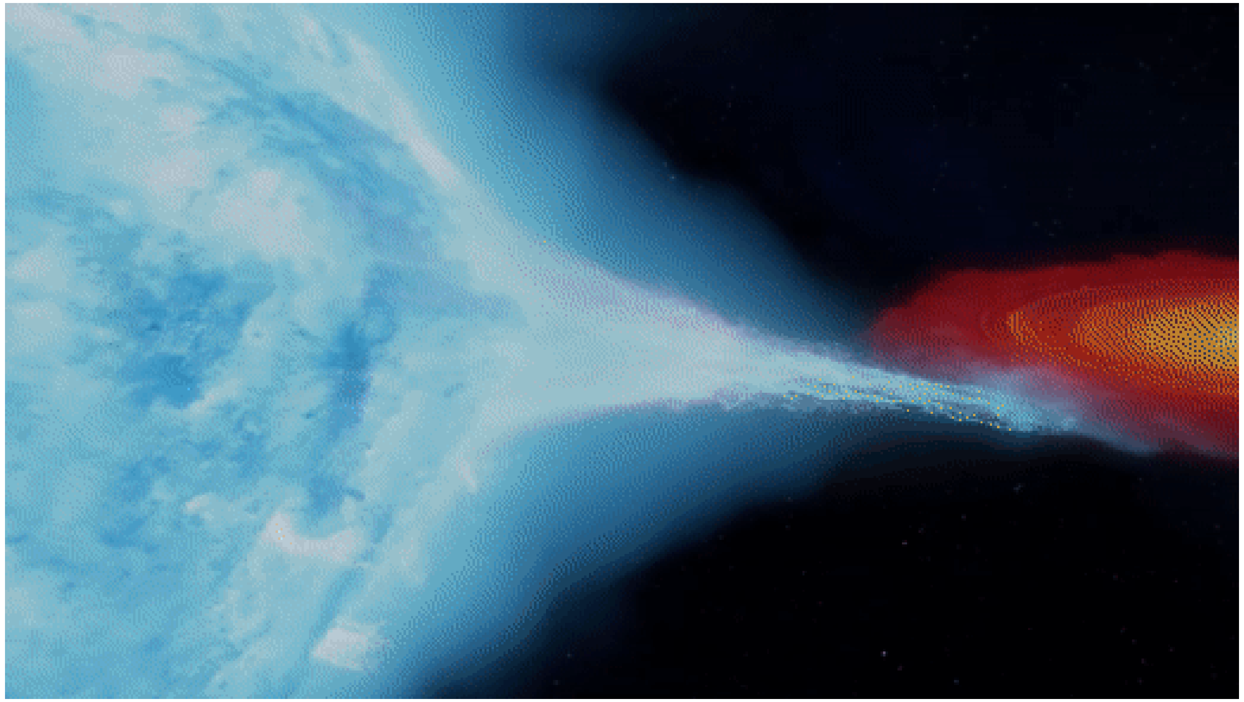
THE UNIVERSITY OF WESTERN AUSTRALIA

Government of Western Australia
Department of the Premier and Cabinet
Office of Science

Twinkle twinkle…

A *transient* is an astrophysical phenomenon whose brightness changes over observable time.

- Supernovae
- Variable stars, e.g., pulsating,
- eclipsing binaries.
- Gamma-ray bursts (GRBs)
- Fast radio bursts (FRBs)
- Transiting planets
- Active galactic nuclei (AGN)
- Accreting blackholes
- and lots more…

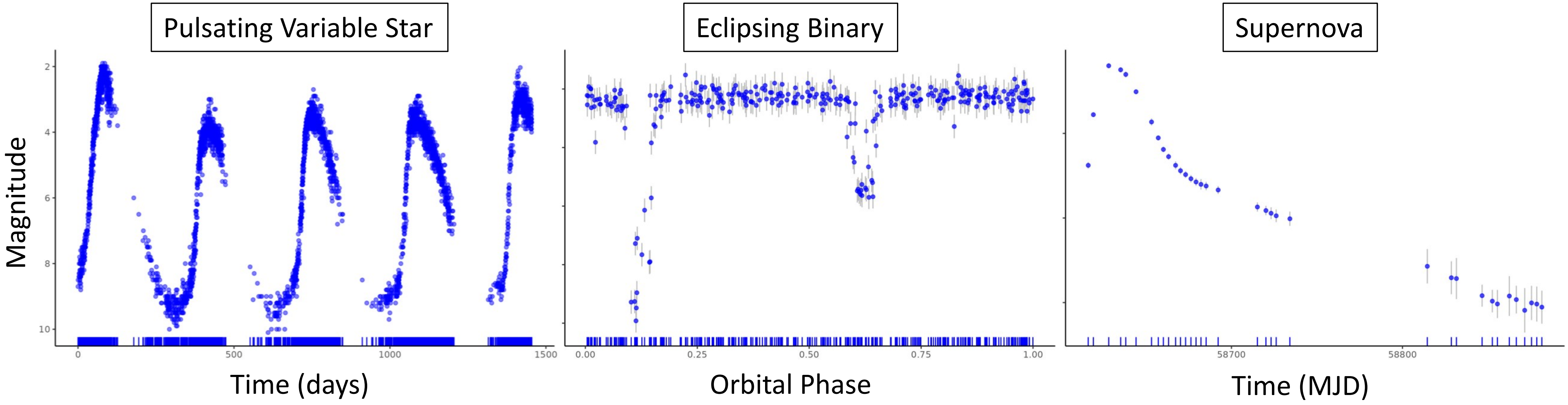


Artist’s impression of the Cygnus X-1 system. Credit: ICRAR

Light Curves

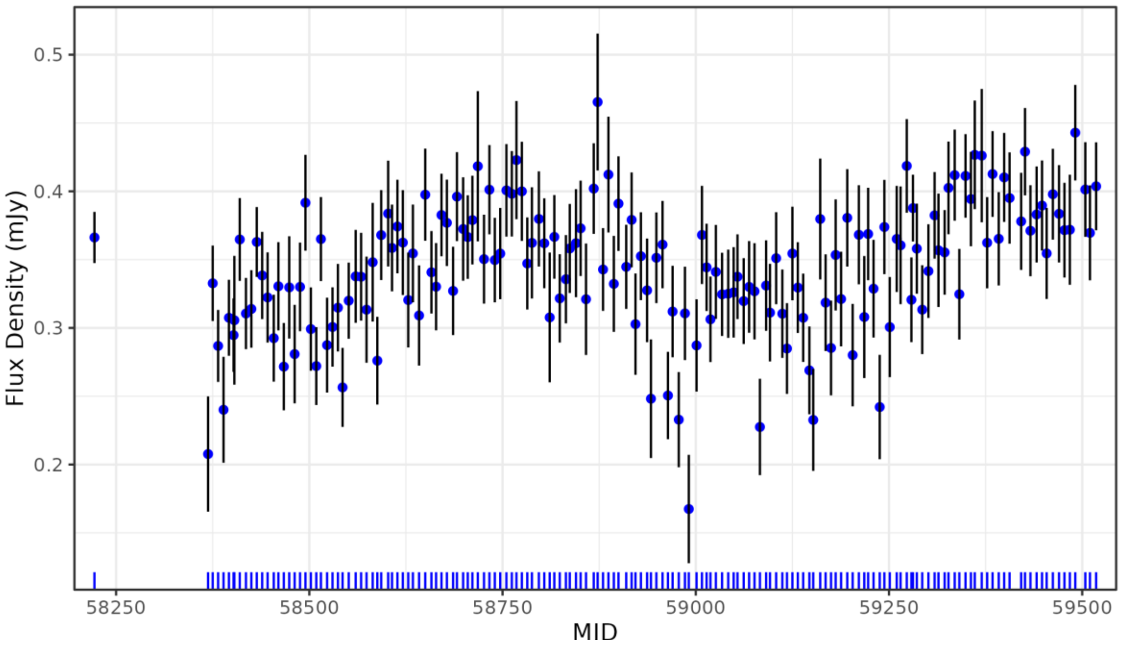
Light curves are time series describing the brightness of a source over time.

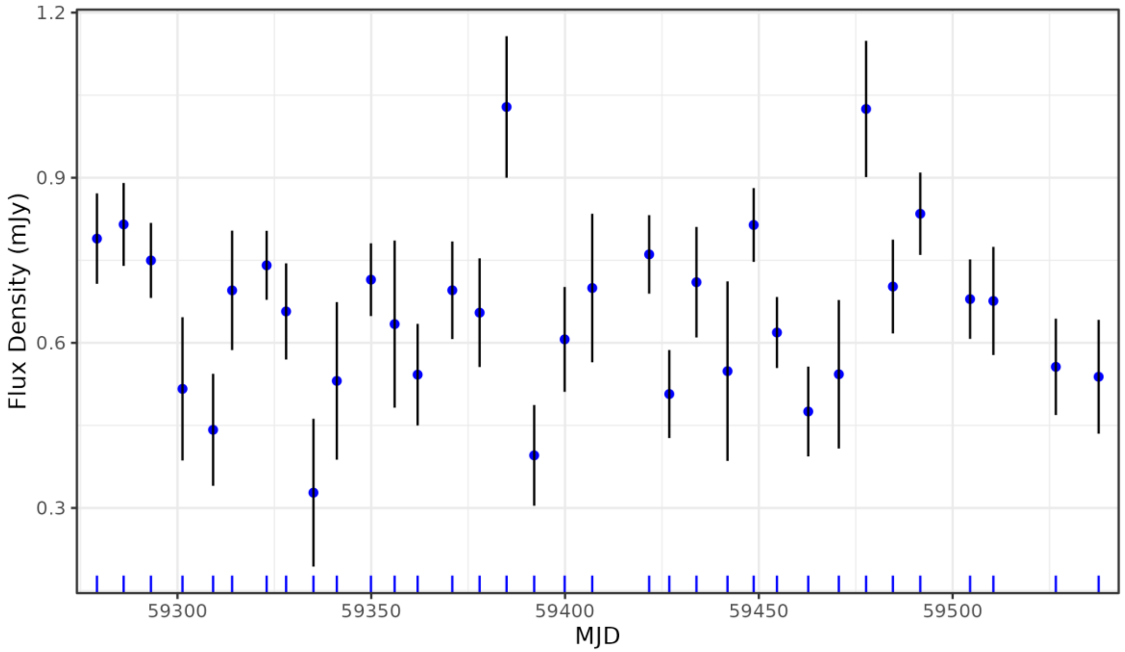
- The shape of a light curve can reveal the type of object or event.
- Variability in brightness can reveal information about the processes underlying the observed phenomenon.

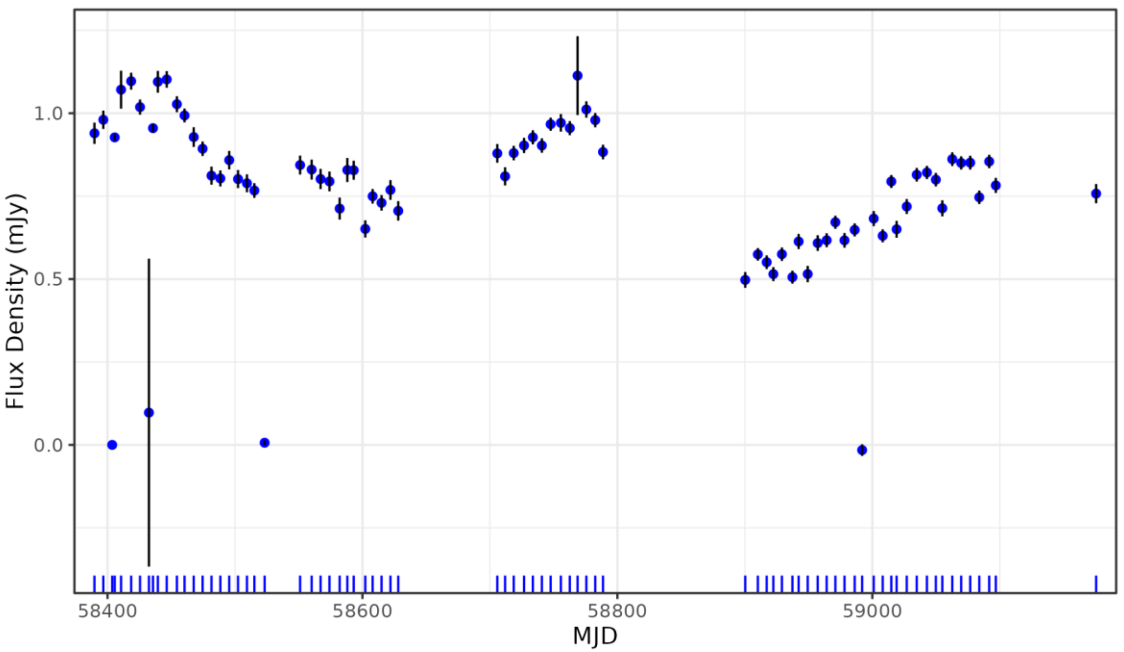


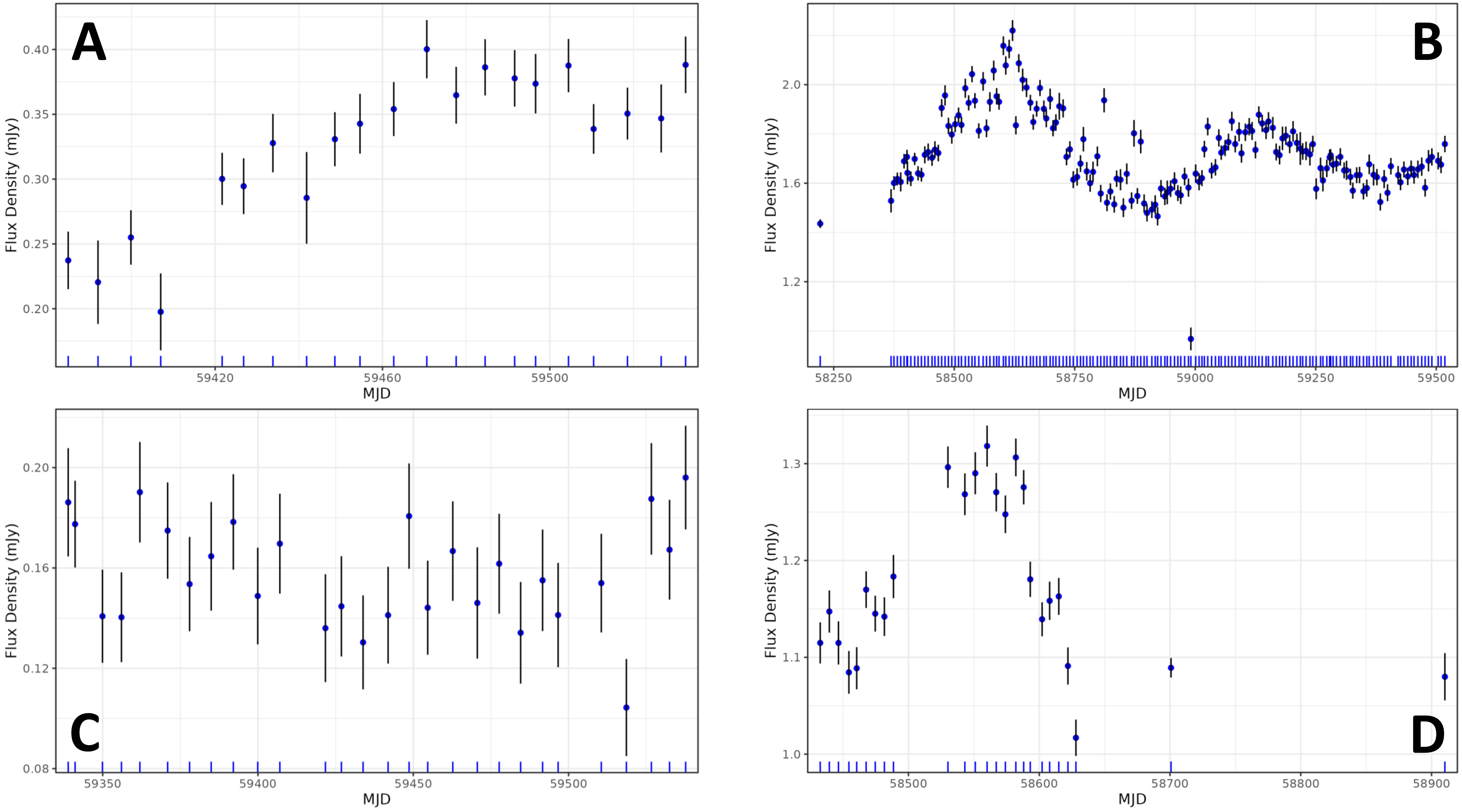
Heterogeneous Data

- Different cadences
- Sparse observations
- Uneven sampling rates
- Varying noise levels








A
B
C
D
Flux Density (mJy)
MJD
0.40
0.35
0.30
0.25
0.20
59420
59460
59500
2.0
1.6
1.2
58250
58500
58750
59000
59250
59500
0.20
0.16
0.12
0.08
59350
59400
59450
59500
1.3
1.2
1.1
1.0
58500
58600
58700
58800
58900

Characterising Light Curves

Oversimplified

- Fewer parameters
- Scales easily
- High information loss

Overspecified

- Many parameters
- High discriminatory power
- Overfitting

Model light curves as a Gaussian Process (GP)

Objectives

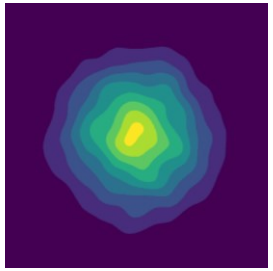
1. Characterisation of light curves based on Gaussian process (GP) regression.
   - Statistically justified and astrophysically meaningful.
2. Identify variable and transient candidates in *large astronomical surveys*, e.g., ThunderKAT, LSST.
3. Guide the astronomical community towards more sophisticated application of GPs to time-series astronomy.

Multivariate Normal $\mathbf{Y} \sim \mathrm{MVN}(\mathbf{0}, \boldsymbol{\Sigma}_{n\times n})$

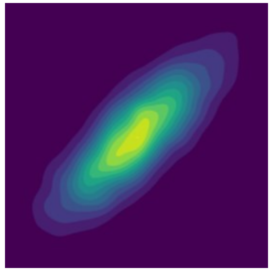
$\boldsymbol{Y}$ is a vector of n Gaussian random variables.

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix} = \boldsymbol{Y} \sim MVN(\boldsymbol{\mu}, \boldsymbol{\Sigma}_{n\times n}), \qquad \boldsymbol{\Sigma}_{n\times n} = \begin{bmatrix} \Sigma_{11} & \cdots & \Sigma_{1n} \\ \vdots & \ddots & \vdots \\ \Sigma_{n1} & \cdots & \Sigma_{nn} \end{bmatrix}$$

where $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_n)$ and $\boldsymbol{\Sigma}$ is a $n \times n$ covariance matrix.



$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\Sigma = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$$

- Symmetric, positive semi-definite matrix.
- Linear combinations of covariance matrices are also valid covariance matrices.

Gaussian Processes

Extend multivariate Gaussian to 'infinite' dimensions.

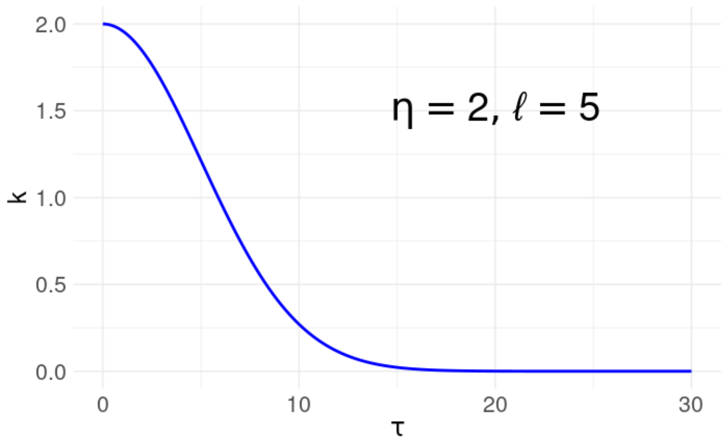
- Mean function, $\mu(t)$
- Covariance or **kernel function**, $\boldsymbol{\kappa}(\boldsymbol{t}, \boldsymbol{t})$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \end{bmatrix} = \boldsymbol{Y} \sim GP(\mu(t), \boldsymbol{\Sigma})$$

where $\boldsymbol{\mu} = \mu(t_i)$ and $\Sigma_{ij} = \boldsymbol{\kappa}(\boldsymbol{t_i}, \boldsymbol{t_j})$, for $i, j = 1, 2, \ldots$

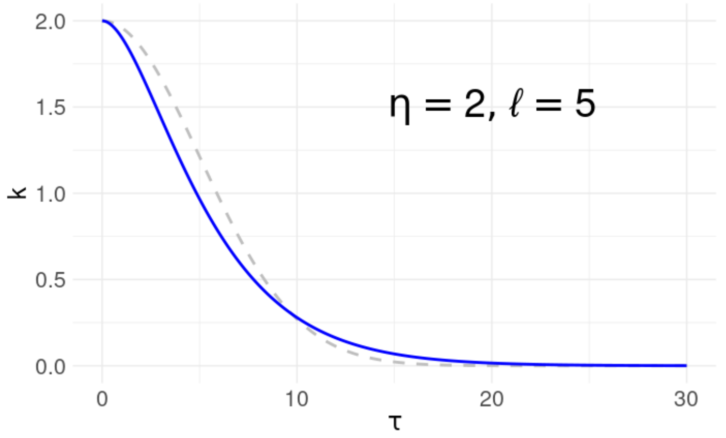
Rather than specifying a fixed covariance matrix with fixed dimensions, compute covariances using the kernel function.

$$\tau = |t_r - t_c|;\ \eta, \ell, T > 0$$



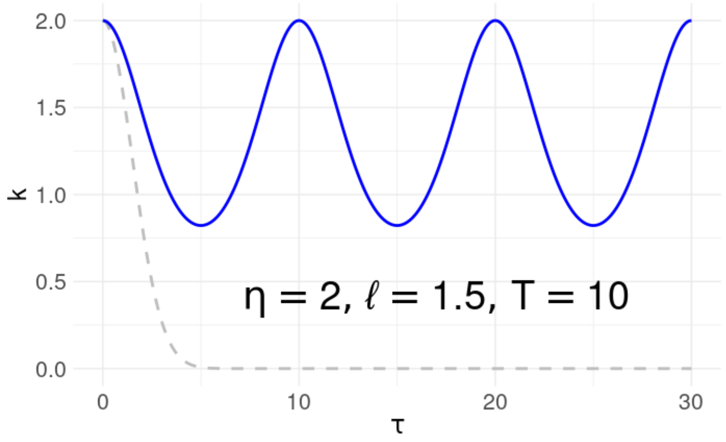
$$\kappa(\tau; \eta, \ell) = \eta \exp\left\{-\frac{1}{2}\left(\frac{\tau}{\ell}\right)^2\right\}$$

Squared Exponential



$$\kappa(\tau; \eta, \ell) = \eta \left(1 + \sqrt{3}\frac{\tau}{\ell}\right) \exp\left\{-\sqrt{3}\frac{\tau}{\ell}\right\}$$

Matern 3/2

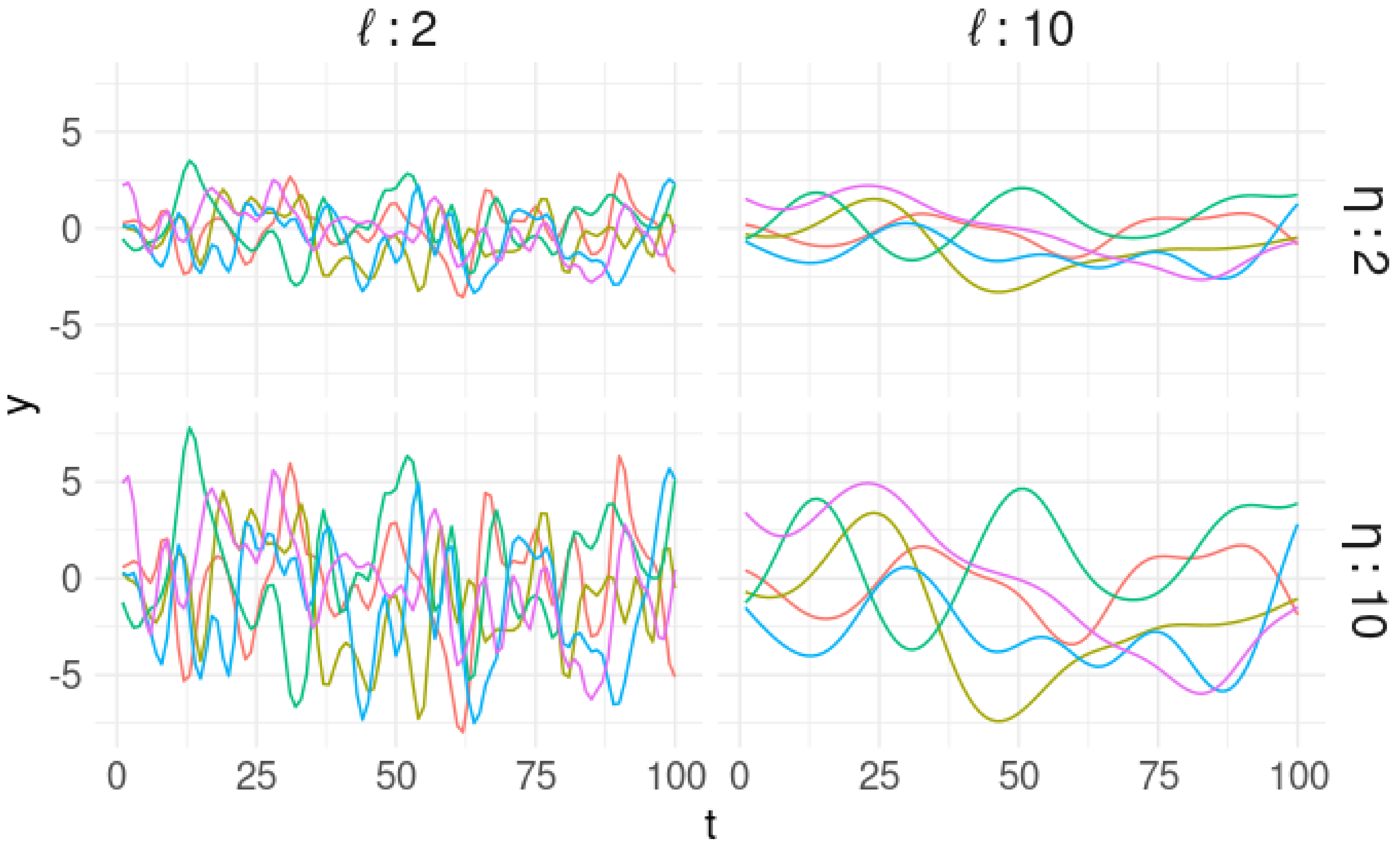


$$\kappa(\tau; \eta, \ell, T) = \eta \exp\left\{-\frac{2}{\ell^2}\sin^2\left(\pi\frac{\tau}{T}\right)\right\}$$

Periodic

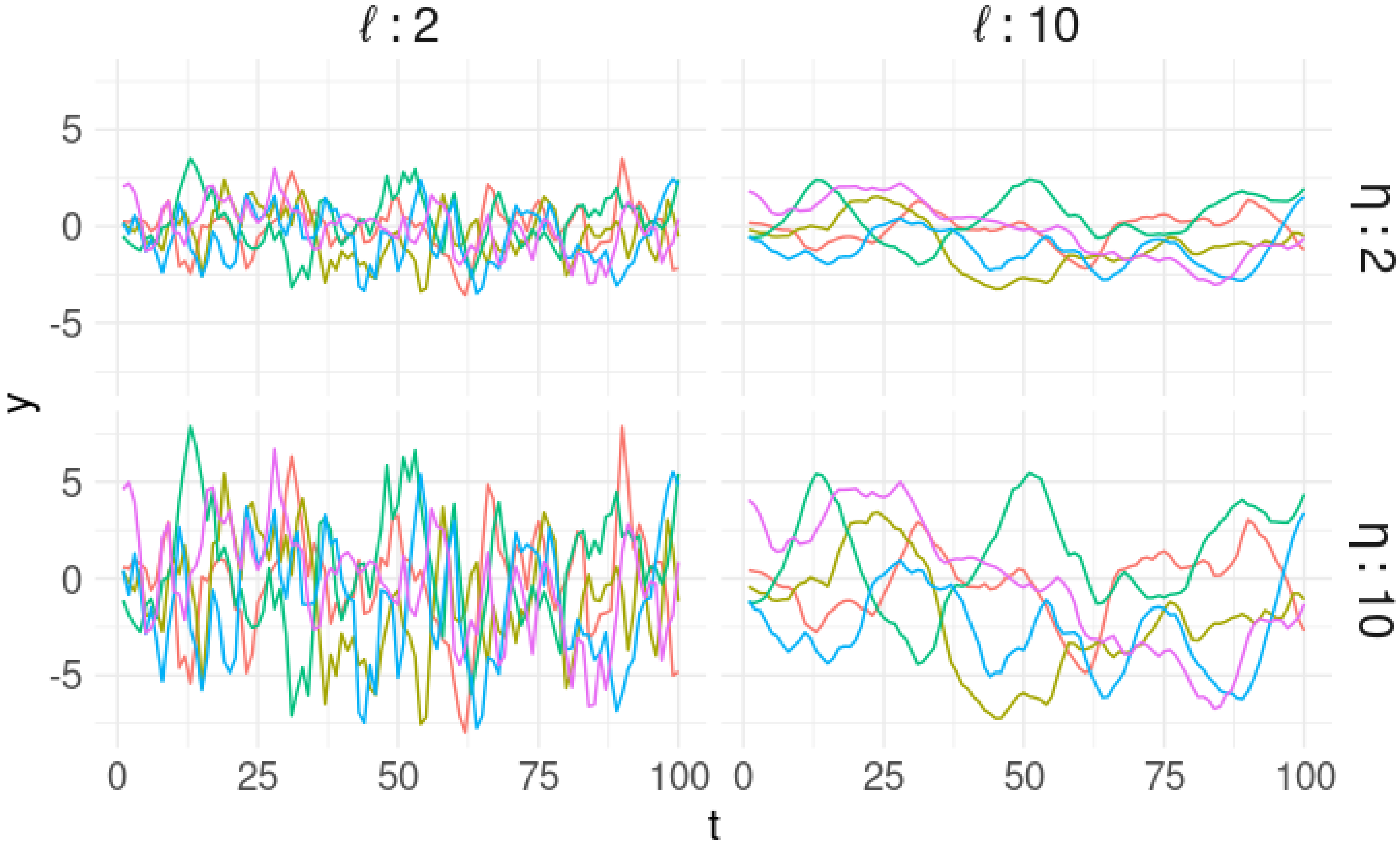
Squared Exponential Kernel

$$\kappa(\tau;\eta,\ell) = \eta \exp\left\{-\frac{1}{2\ell^2}\tau^2\right\}$$



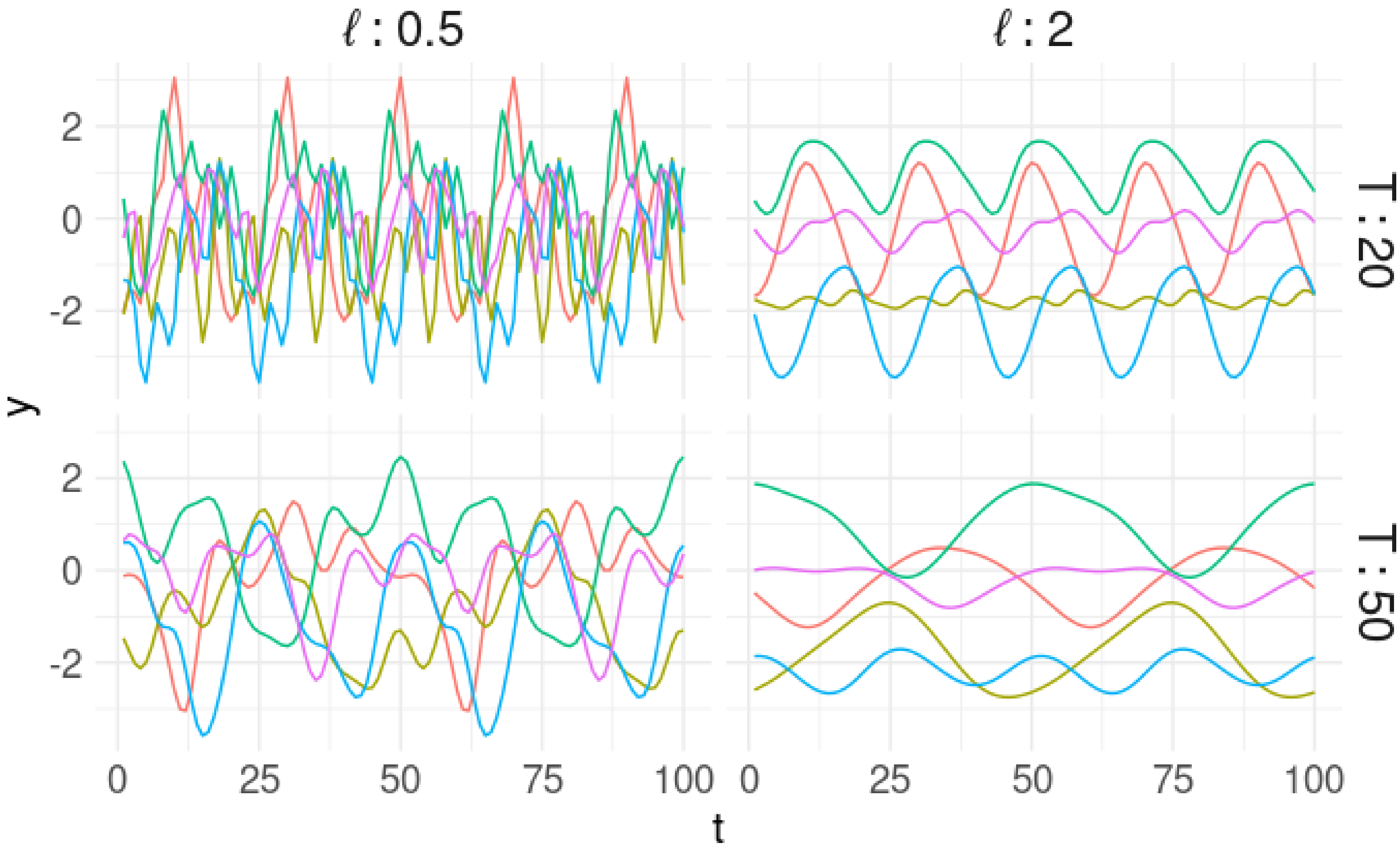
Matern 3/2 Kernel

$$\kappa(\tau;\eta,\ell) = \eta\left(1+\sqrt{3}\frac{\tau}{\ell}\right)\exp\left\{-\sqrt{3}\frac{\tau}{\ell}\right\}$$



Periodic Kernel

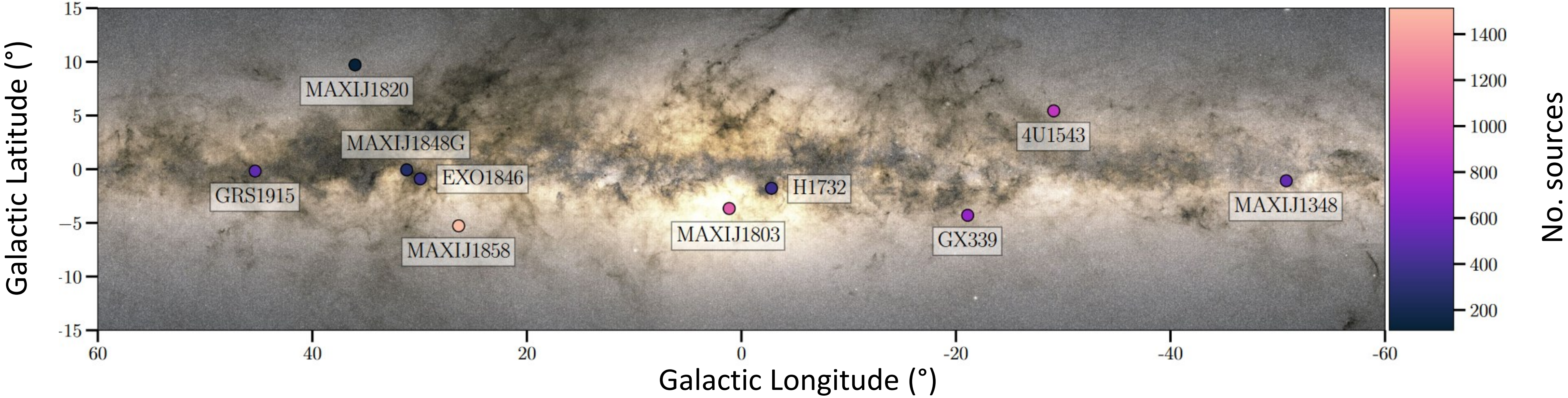
$$\kappa(\tau;\eta,\ell) = \eta \exp\left\{-\frac{2}{\ell^2}\sin^2\left(\pi\frac{\tau}{T}\right)\right\}$$



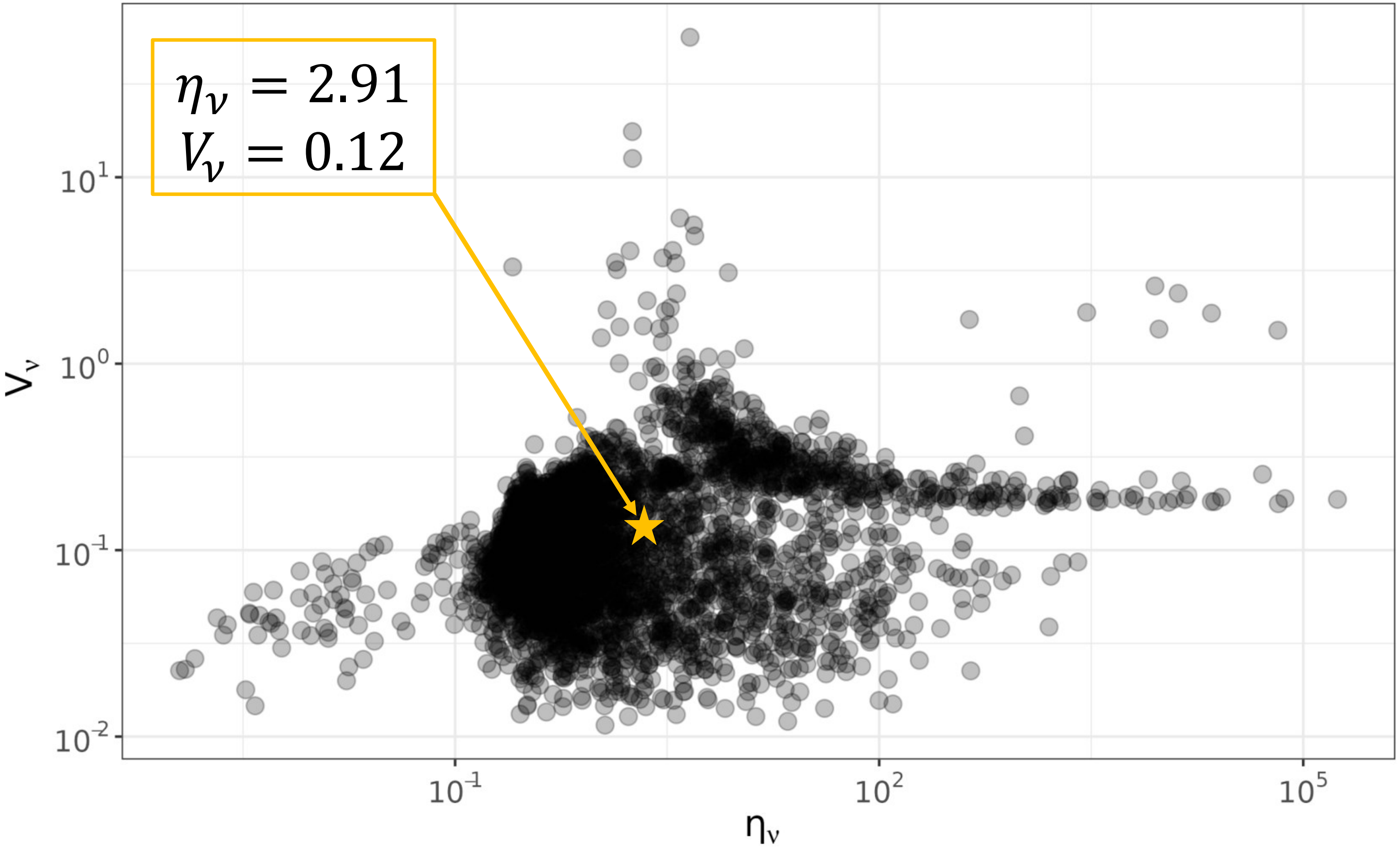
ThunderKAT Survey

- Image-domain transients survey using MeerKAT
- Field of view of ≈ 1 square degree
- 6,394 radio light curves over 10 fields
- Flux density measurements
- Standard errors

MeerKAT Radio Telescope (Credit: SARAO)



Variability Statistics: η_ν and V_ν



(Data courtesy of Andersson, 2023)

$$\eta_\nu = \frac{1}{N}\sum\left(\frac{\text{Obs.} - \text{Wt. Mean}}{\text{Std. Error}}\right)^2$$

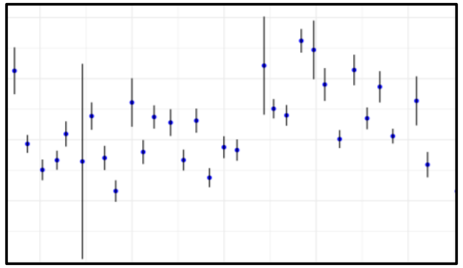
$$\sim \chi^2_{N-1}$$

$$V_\nu = \frac{\text{Standard Deviation}}{\text{Mean}}$$

As $\eta_\nu \to \infty$ and $V_\nu \to \infty$

Source is likely transient

Modelling Workflow



Data → Likelihood (Gaussian)

Flux Densities

GP Prior

GP Posterior

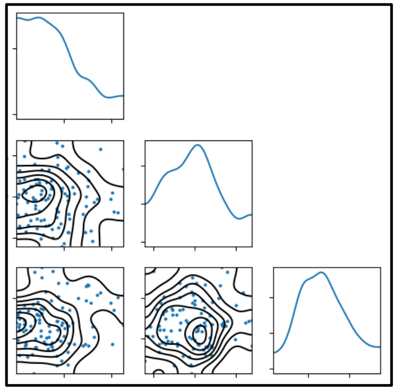
$\mu(t) = 0$ Mean Function

Kernel Function

SE
Matern 3/2
Periodic

$\eta \sim \mathrm{HalfNormal}()$
$\ell \sim \mathrm{InvGamma}()$
$T \sim \mathrm{Uniform}()$

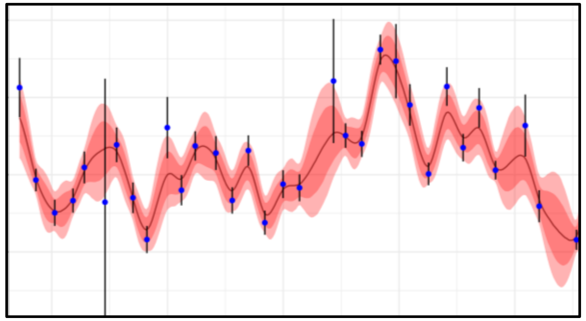
Hyperparameter priors

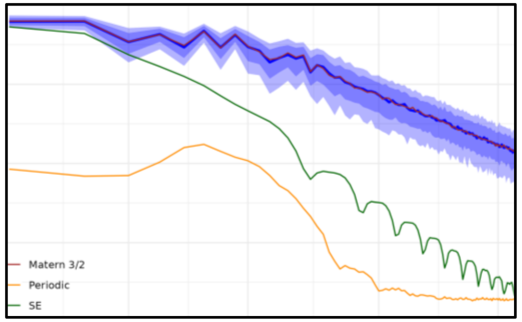


Hyperparameter posteriors

$$\boldsymbol{\theta} = (\boldsymbol{\eta}, \boldsymbol{\ell}, \boldsymbol{T})$$

Characterisation of Light Curve





Posterior Predictive

Power Spectral Density (PSD)

ThunderKAT GP Model

$$Y \sim \text{MVN}(f, \boxed{\hat{\boldsymbol{e}}^2 \boldsymbol{I}})$$ Gaussian White Noise

Latent Function $f \sim \boxed{\text{GP}(\mathbf{0}, \boldsymbol{K}_{N\times N})}$ GP Prior

$$\boldsymbol{K}_{rc} = \kappa(t_r, t_c | \boldsymbol{\theta})$$

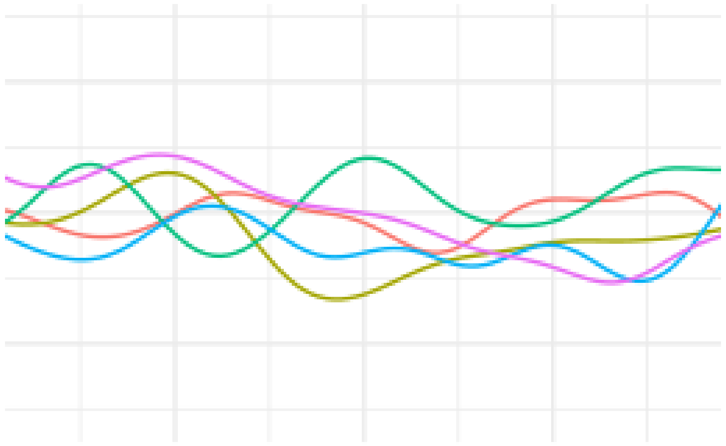
$$= \boxed{\kappa_1(\tau; \eta_{SE}, \ell_{SE})} + \boxed{\kappa_2(\tau; \eta_{M32}, \ell_{M32})} + \boxed{\kappa_3(\tau; \eta_P, \ell_P, T)}$$

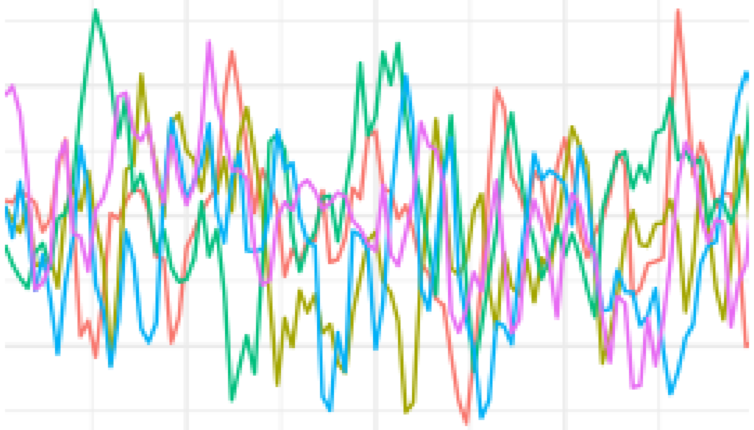
Squared Exponential — Matern 3/2 — Periodic

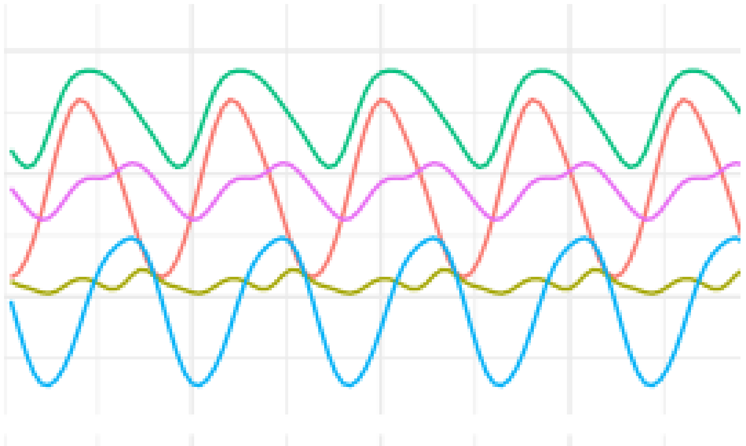
Covariance Kernel

$$r, c = 1, \ldots, N.$$

$$\boldsymbol{\theta} = (\eta_{SE}, \ell_{SE}, \eta_{M32}, \ell_{M32}, \eta_P, \ell_P, T)$$







Hyperparameter Priors

$$\eta_{SE}, \eta_{M32}, \eta_P \sim N^+(0, 1)$$

Fitting to standardised flux densities

$$\ell_{SE}, \ell_{M32}, \ell_P \sim \text{InverseGamma}\left(\alpha = 3, \beta = \frac{1}{2}\text{range}(t)\right)$$

No more than half of total duration

$$T \sim \text{Uniform}\left[2 \times \min(\Delta t),\ \frac{1}{4}\text{range}(t)\right]$$

Observe at least four cycles of any periodicity

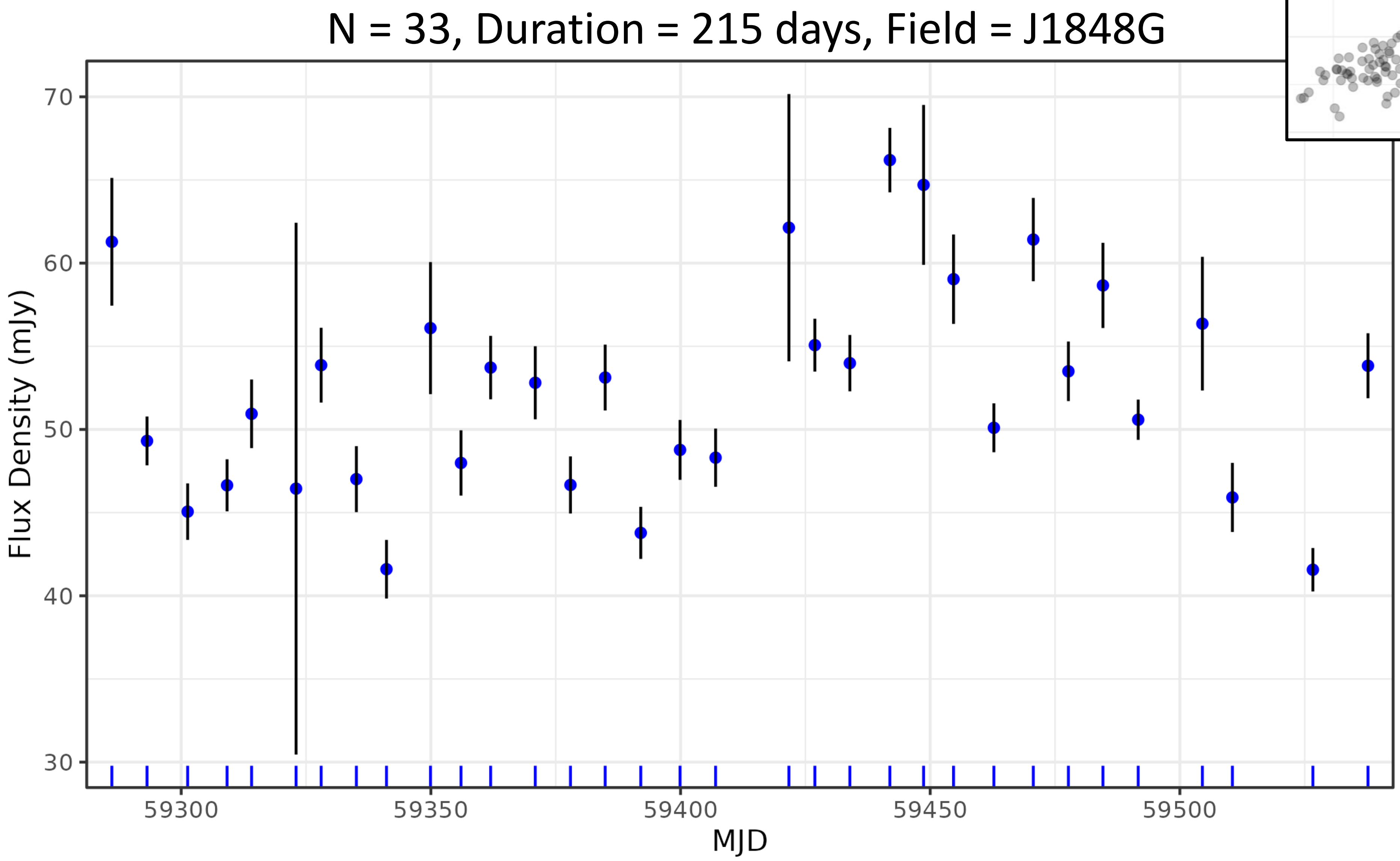
$$\ell_{M32} < \ell_{SE}$$

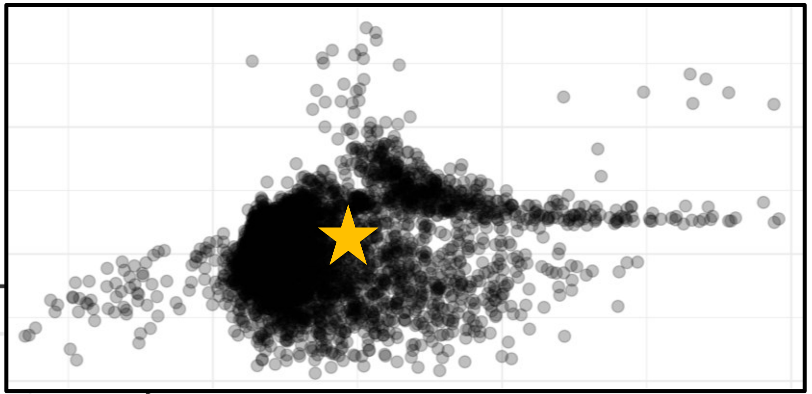
Bias SE kernel towards fitting longer term smooth trends

$$\min(\Delta t) < \ell_{.}$$

Constrain length scale to be at least as wide as the narrowest gap in light curve

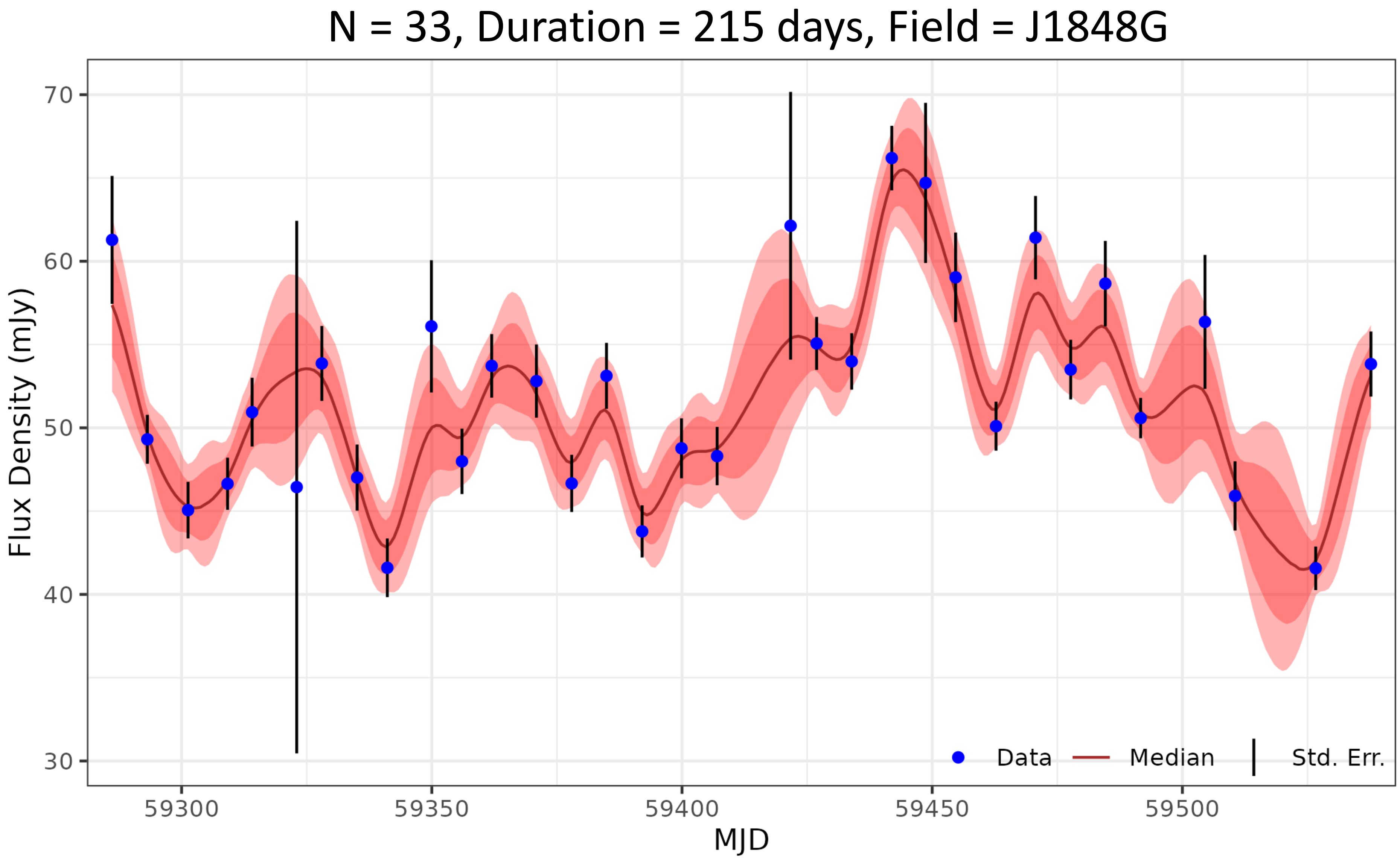
GP Fitting Example





$\eta_\nu = 2.91$
$V_\nu = 0.12$

Posterior Predictive Samples



$\eta_{SE} = 0.39$

$\boldsymbol{\eta_{M32} = 1.26}$

$\eta_P = 0.50$

$\ell_{SE} = 50.0$

$\boldsymbol{\ell_{M32} = 11.9}$

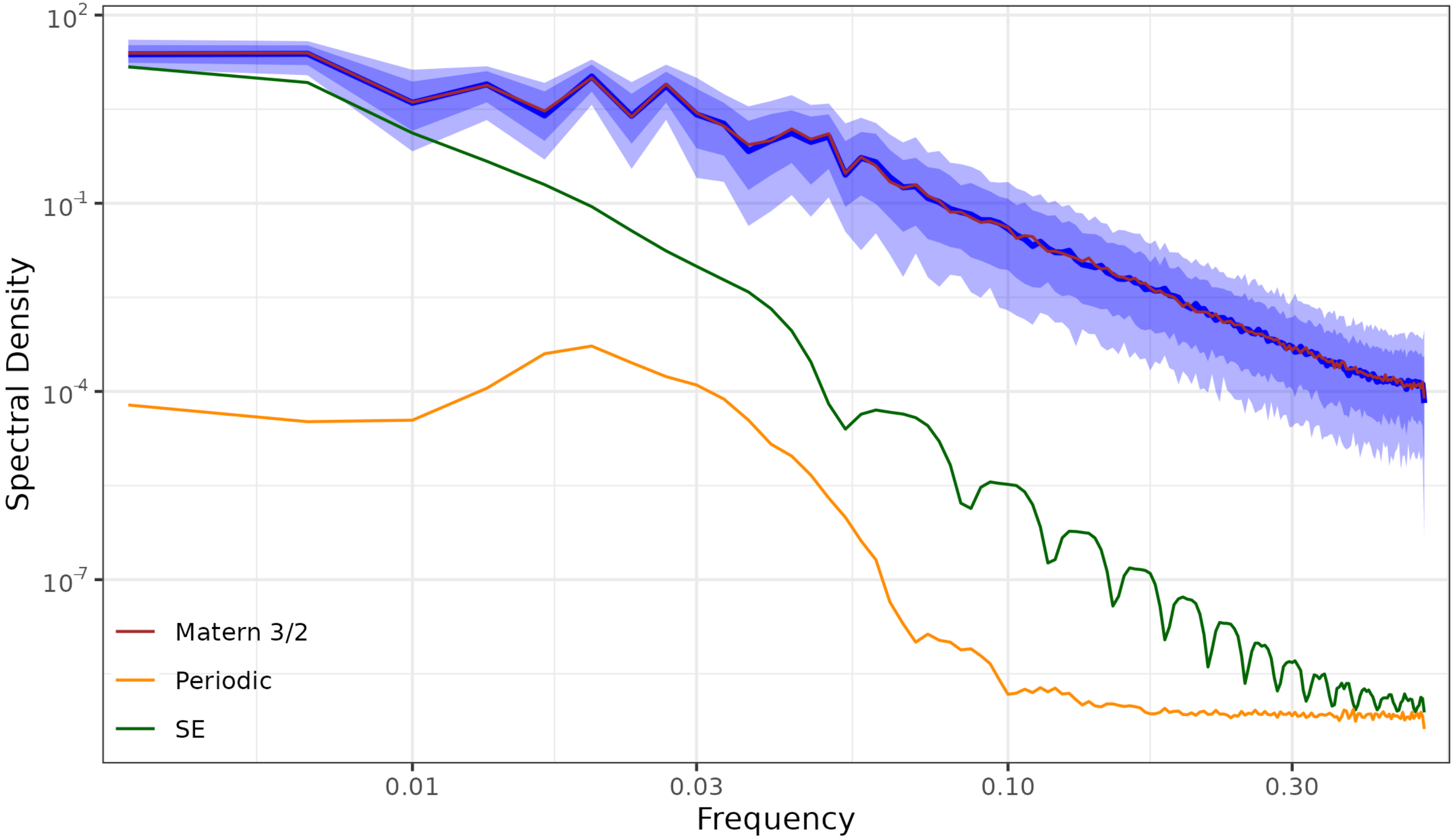
$\ell_P = 46.7$

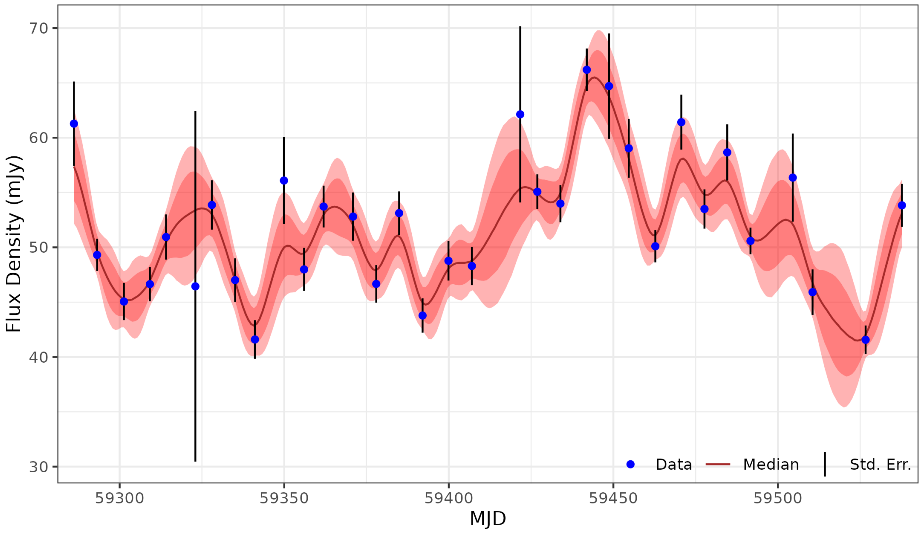
$T = 41.1$

$\eta_\nu = 2.91$

$V_\nu = 0.12$

Power Spectral Density

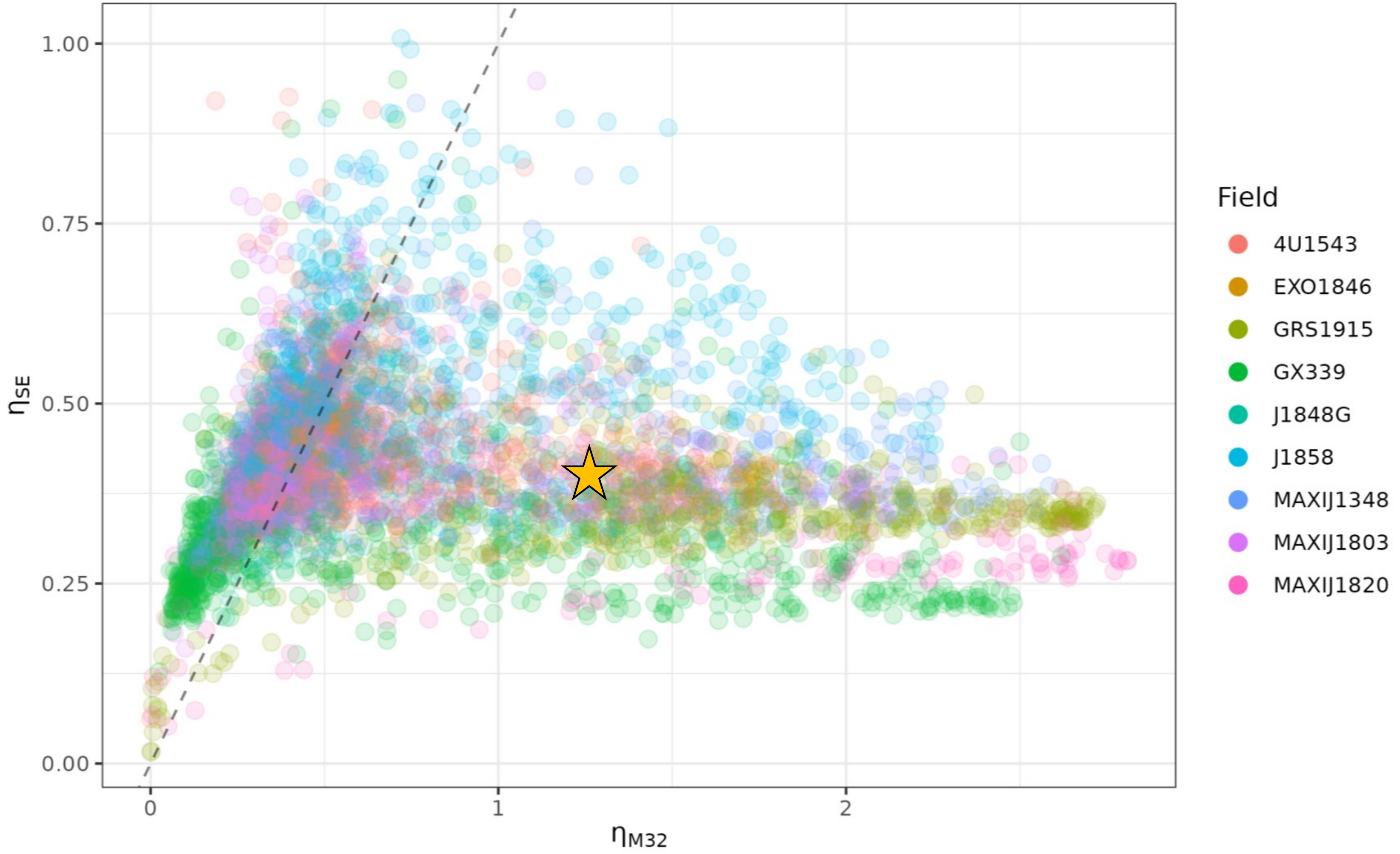


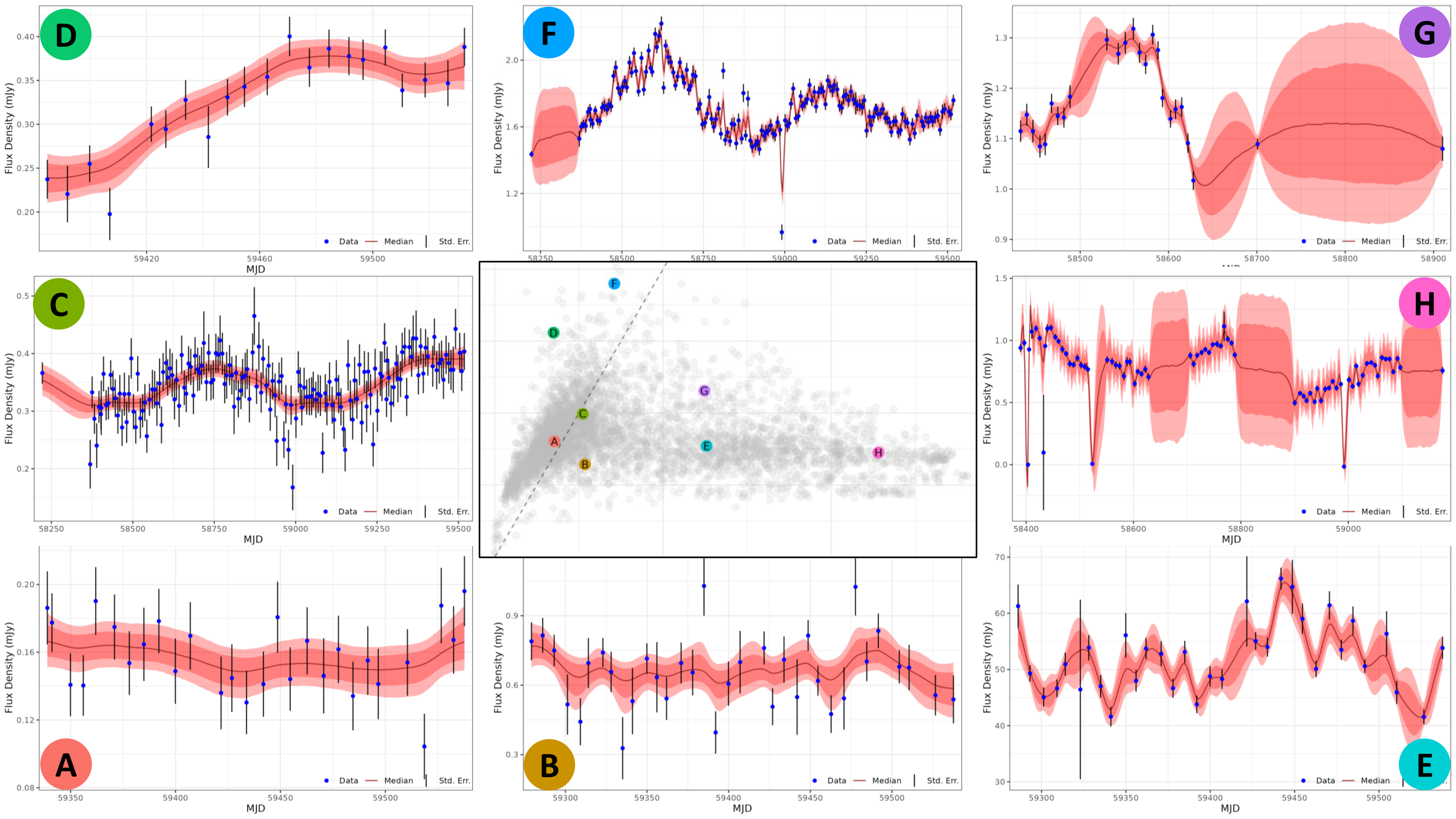


- Matern term dominates
- Very weak periodic term

Large Amplitude ↔ Transience

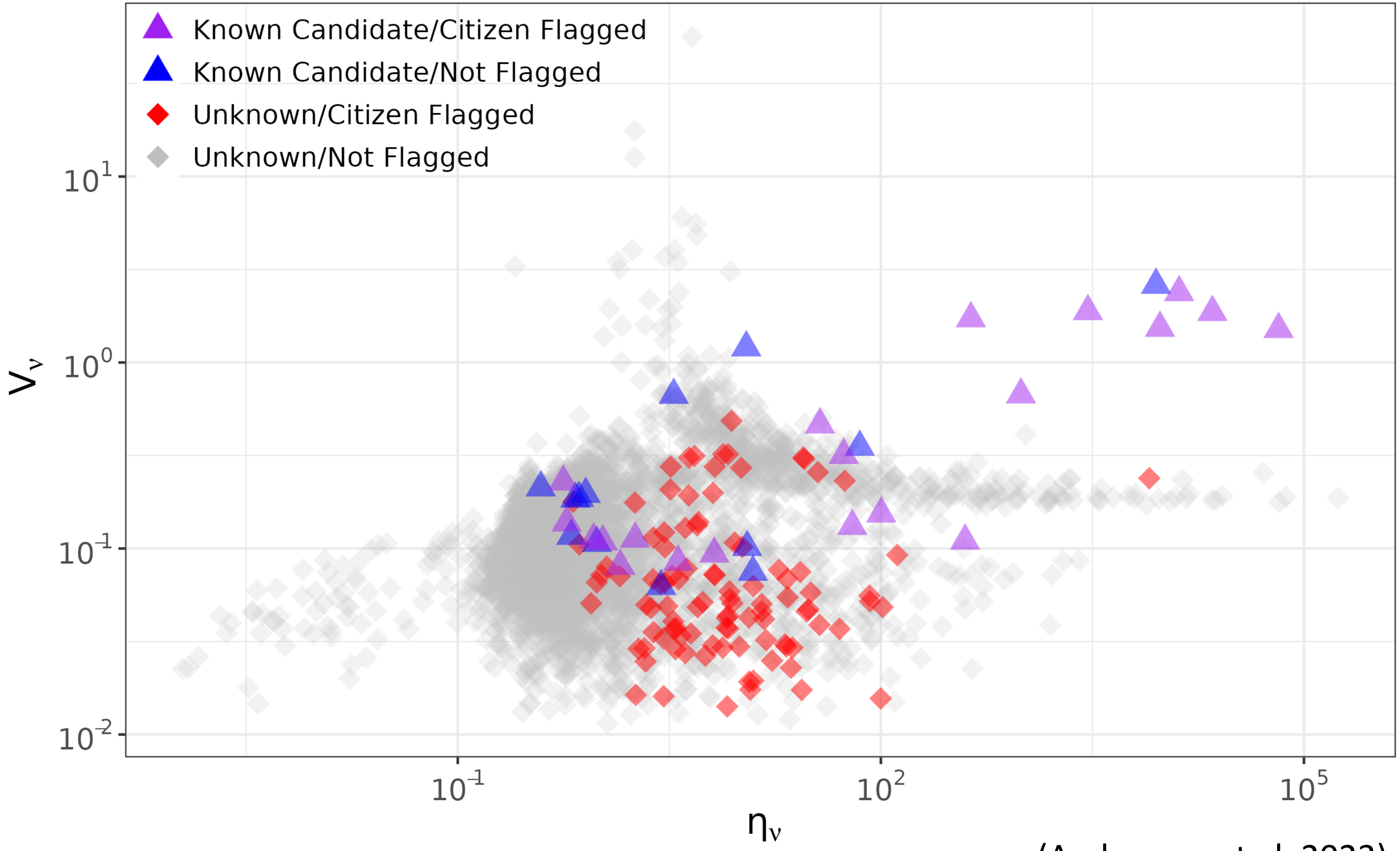
- Brightness is encoded in the **amplitude**, η.
- Transience is characterised by large changes in brightness.





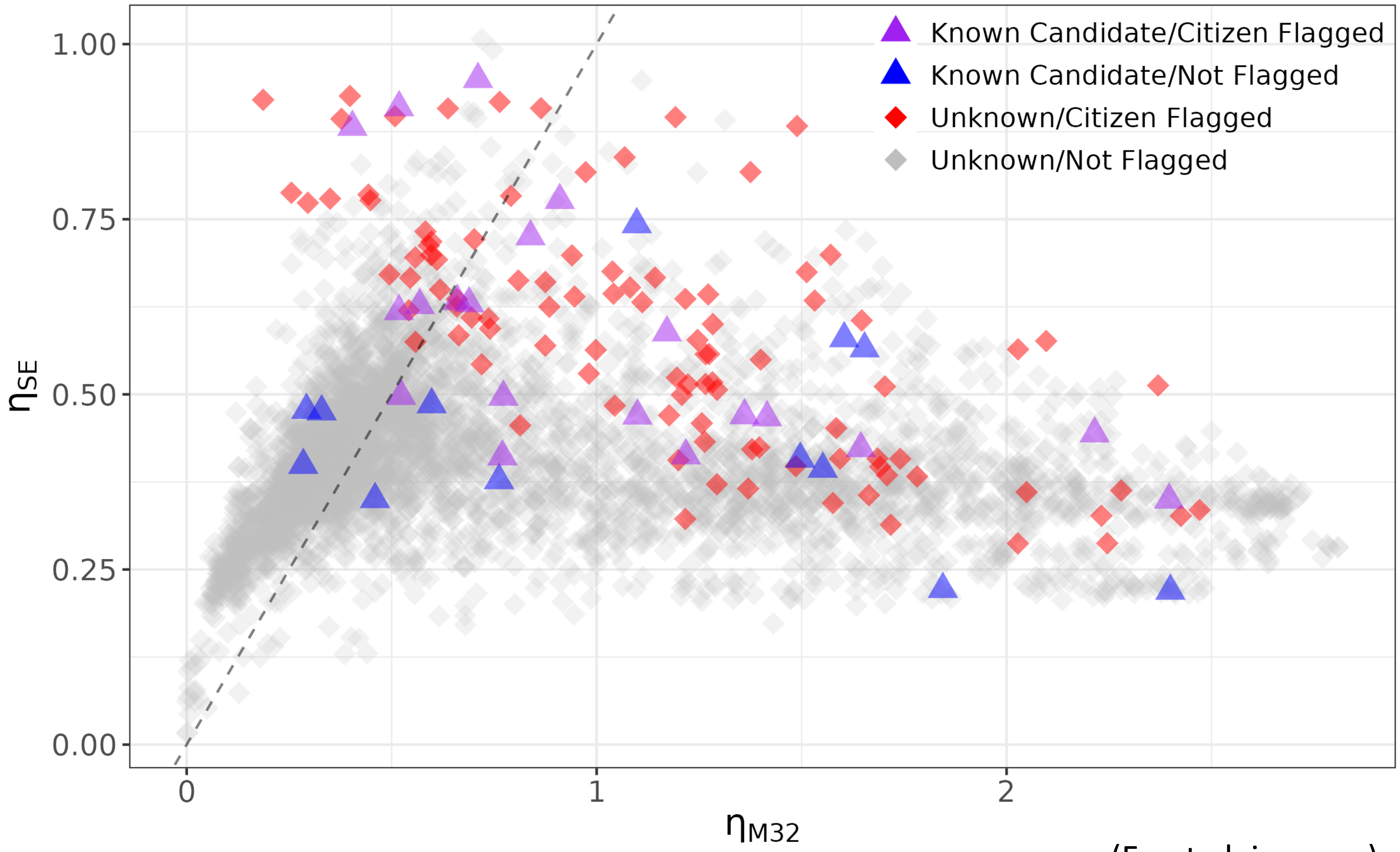
D
F
G
C
H
A
B
E
Flux Density (mJy)
MJD
Data
Median
Std. Err.

Comparison with V_ν vs η_ν



(Andersson et al. 2023)

Comparison with V_ν vs η_ν



(Fu et al. in prep.)

Tools

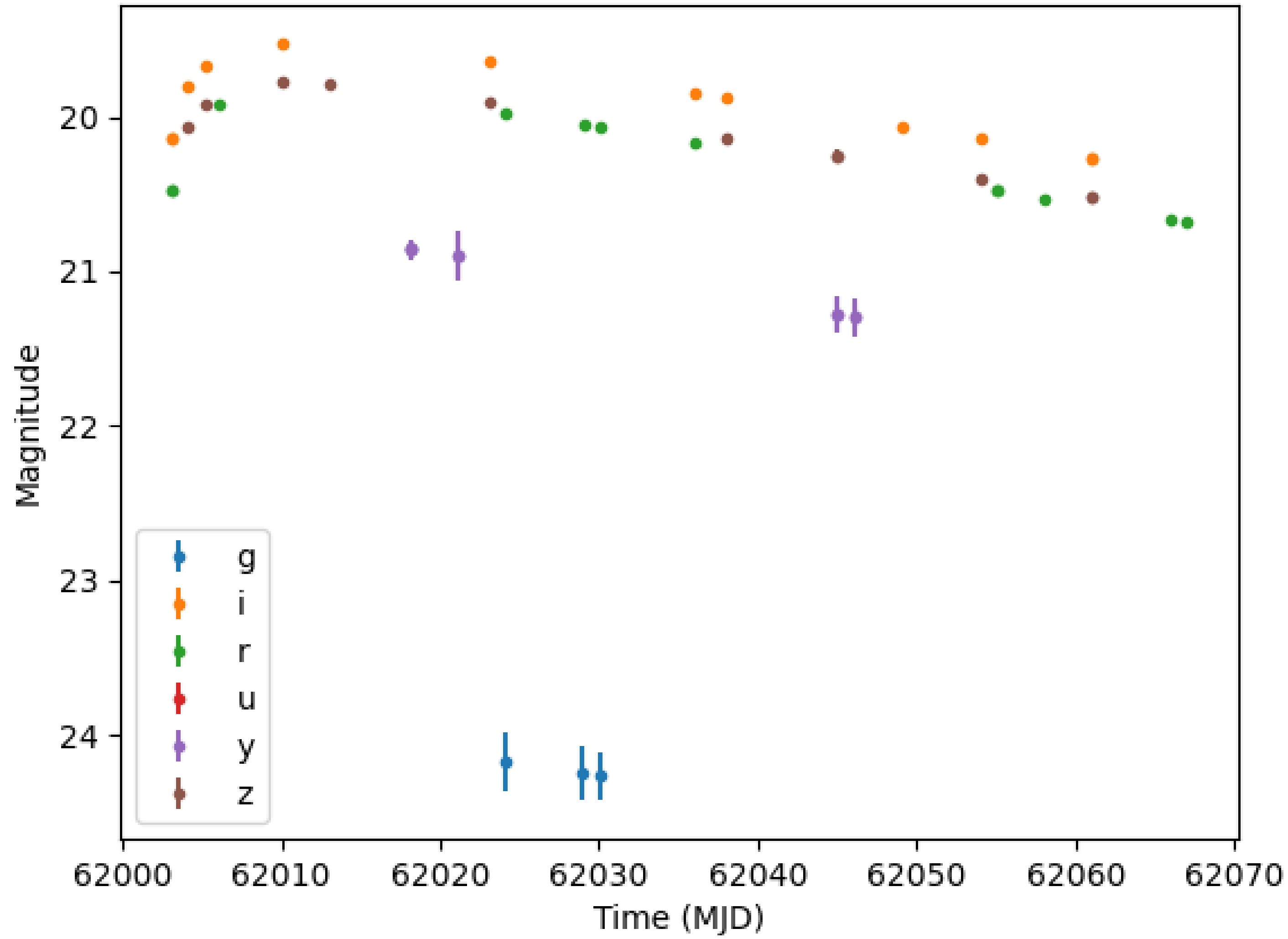
- Implemented in `Python`[1] (v3.10) and `PyMC`[2] (v3.5.2)
  - Accessible to astronomers
  - Probabilistic programming framework
  - Well-maintained open-source software
- Repeated analyses in `R`[3] (v4.3.1) and `Stan`[4] (v2.34)
- Also considered: `celerite2`[5], `george`[6].

1. https://www.python.org
2. https://www.pymc.io
3. https://cran.r-project.org/
4. https://mc-stan.org/
5. https://celerite2.readthedocs.io/en/latest/
6. https://george.readthedocs.io/en/latest/

Outcomes So Far

- New metric for describing the “transience” of light curves.
- Applied this to ThunderKAT to get candidate transients.
- Implemented these models in R/Stan and Python/PyMC for sharing with user community.
- Conducted sensitivity experiments: increasing sparsity, regular vs irregular sampling, permutation tests.
- Manuscript is almost complete.

Multi-band Optical Light Curves



(Credit: S. Sett, 2024)

- LSST light curves may have measurements in multiple bands.
- Expect each band to be correlated.
- Sparsity and sampling will differ between bands.
- Multivariate GPs with different noise model.

Summary

- Developed models and code suitable for fitting univariate GPs to the light curves of a large radio survey, i.e., ThunderKAT.
- GP amplitude hyperparameters are a better descriptor of variability than more commonly used statistics.
- Gaussian processes should be used to perform inference as well as interpolation in time-domain astronomy.
- Extend into multi-band multi-variate GPs for optical light curves and non-Gaussian likelihoods for X-ray light curves.

GPs: not only a means to an end but an end to only means.