

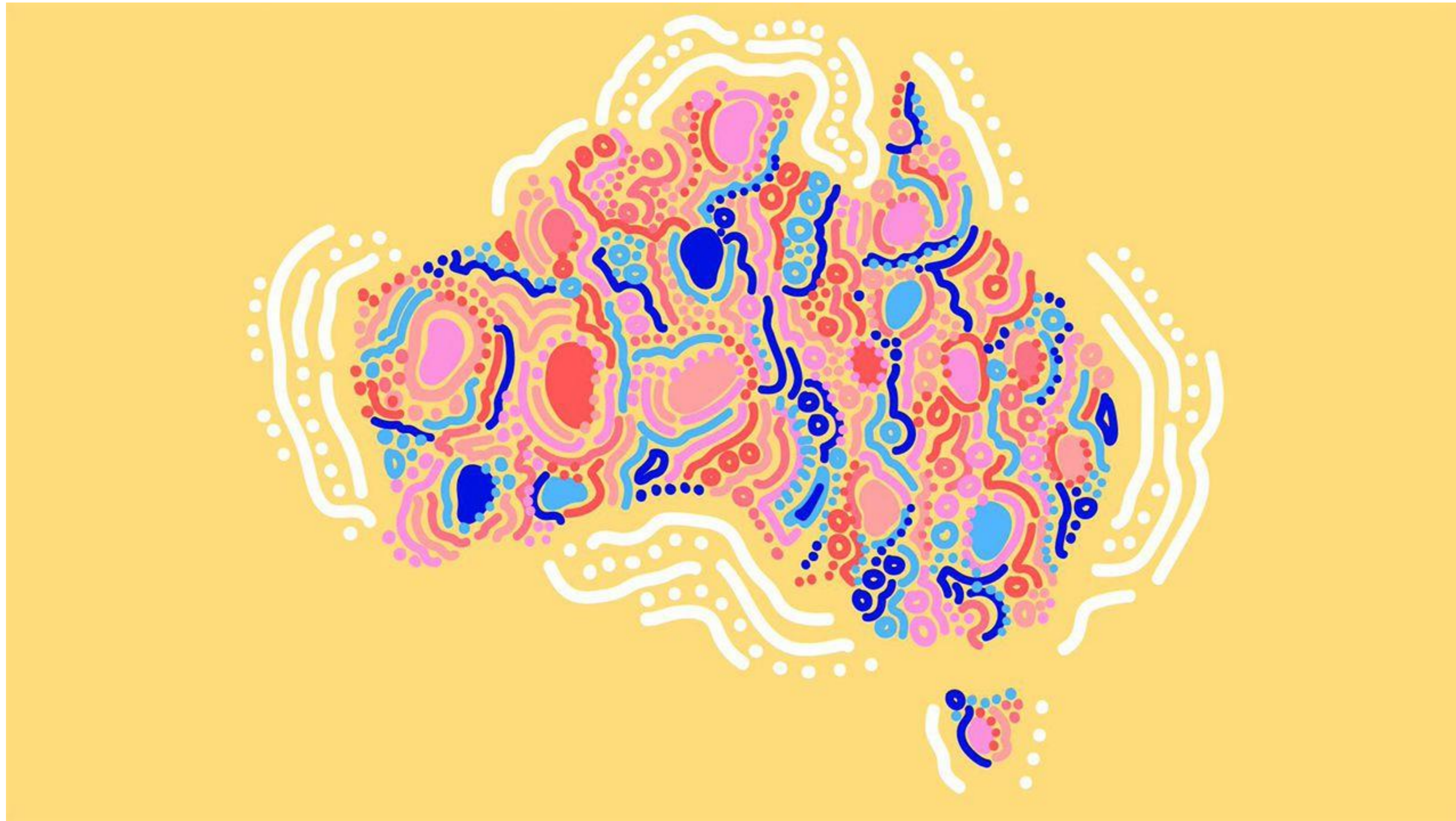
Essential Statistical Concepts for Understanding the COVID-19 Pandemic

Shih Ching Fu

30 March 2022



Acknowledgement of Country



Aim

Provide an entry-level glossary of statistical concepts that are frequently encountered in academic and popular literature on COVID-19.

Who am I?

- Freshly minted biostatistician
- Based at the Clinical Trials Enablement Platform WA (CTEP-WA) at Curtin University.
- Funded by Western Australian Health Translation Network (WAHTN) Biostatistician Fellowship.
- Not an expert in infectious disease modelling, epidemiology, or COVID-19!



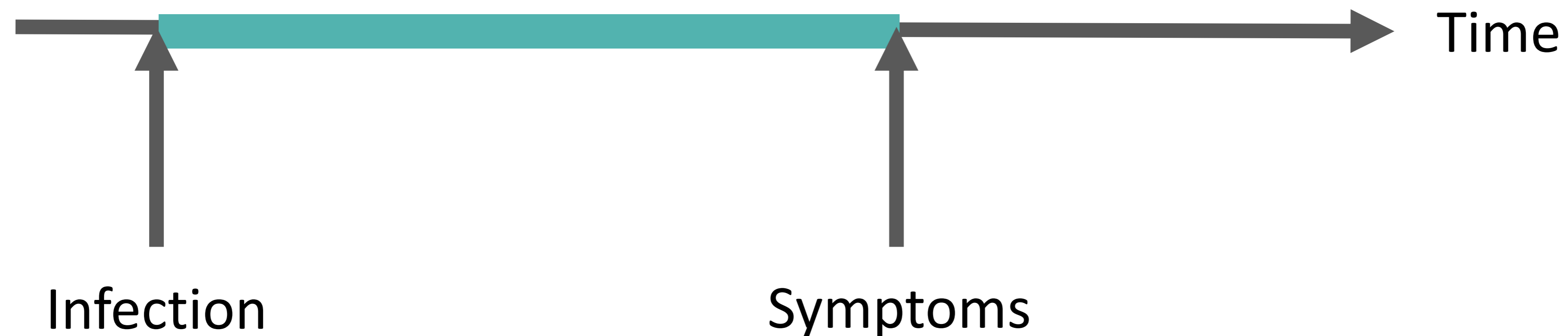
Outline

- Virus Transmission
- Mortality
- Testing
- Statistical Inference
- Q & R

Virus Transmission

Incubation Period

- Incubation period is the interval from receipt of infection to the time of onset of clinical illness, i.e., the onset of recognisable symptoms.
- Australia's national COVID-19 public health guidelines use a **14-day** incubation period to inform many public health measures, such as quarantine and isolation.



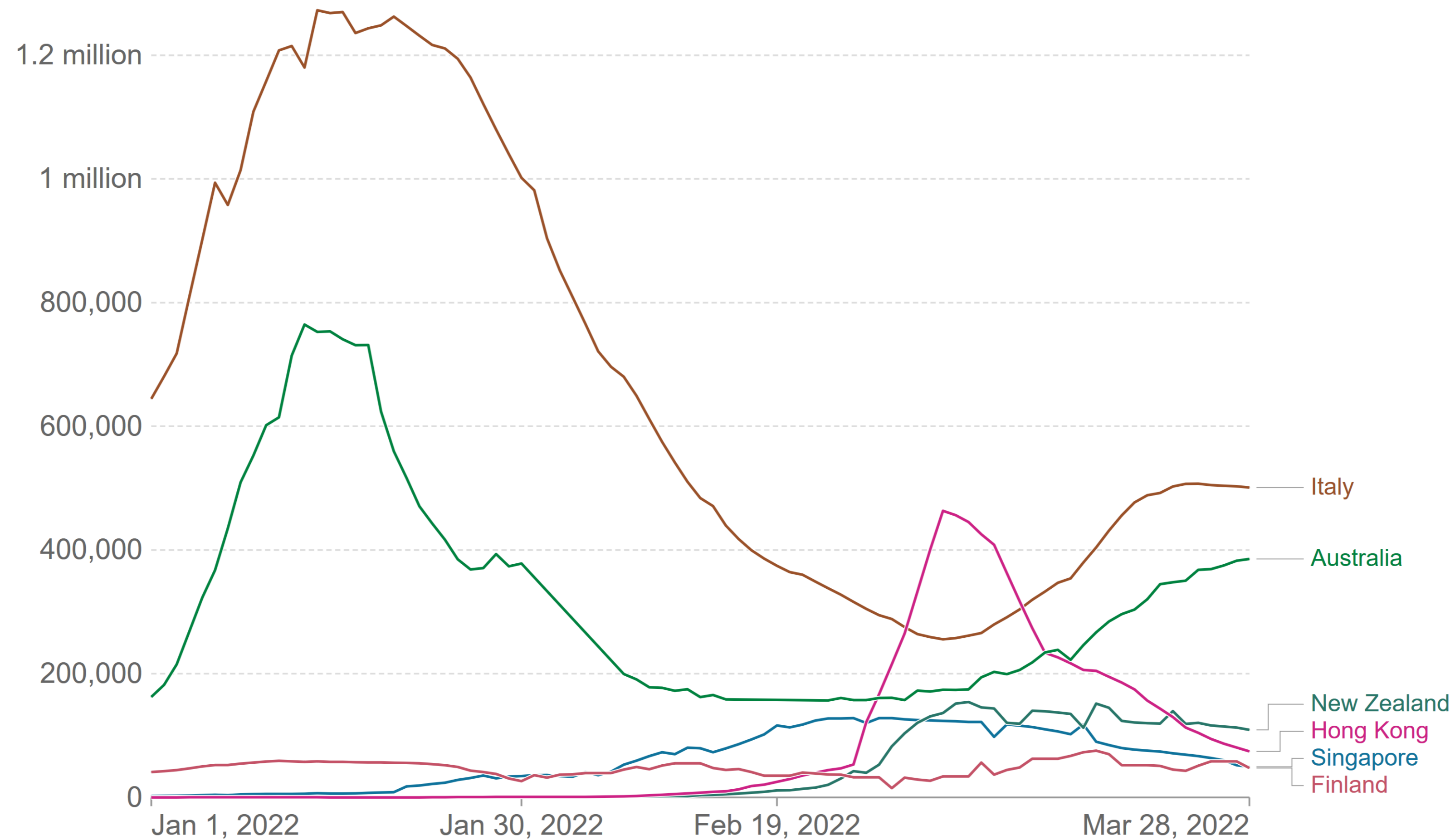
Epidemic Curve

- Epidemic curves depict the progression of an outbreak over time.
- Shows the distribution of the times of onset of disease (incubation period).
- Typically there is a delay between start of illness and reports to public health authorities.

Example: Epidemic Curve

Weekly confirmed COVID-19 cases

Weekly confirmed cases refer to the cumulative number of confirmed cases over the previous week.



Basic Reproductive Number (R_0)

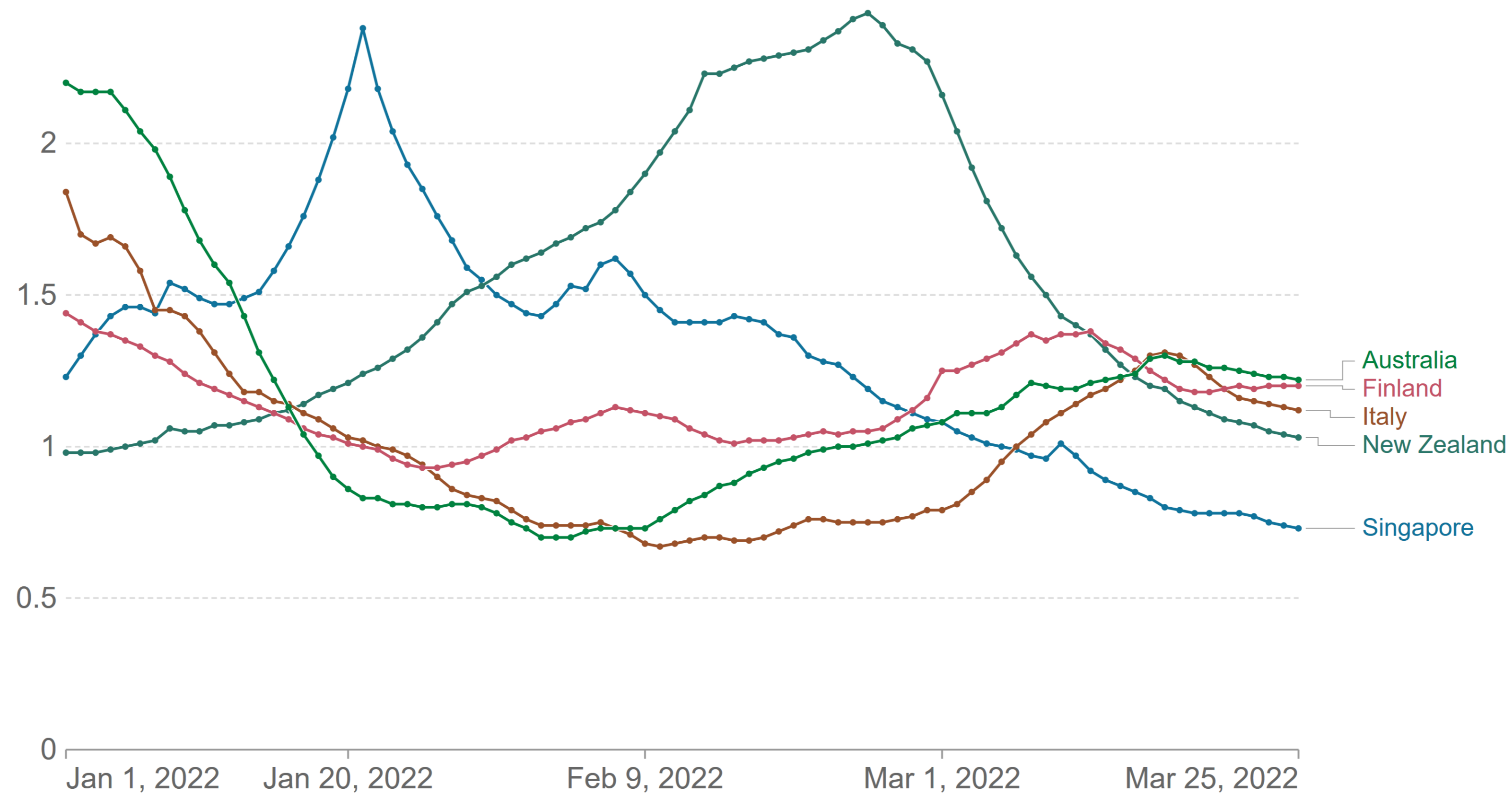
Expected number of additional cases that one case will generate, on average, over the course of its infectious period in an otherwise uninfected population.

- Until this number falls below 1.0, it is likely that an outbreak will continue to spread.
- In January 2020 the R_0 for COVID-19 was estimated to be 2.2 (Li, Guan, Wu, et al. 2020).

Example: R_0

Estimate of the effective reproduction rate (R) of COVID-19

The reproduction rate represents the average number of new infections caused by a single infected individual. If the rate is greater than 1, the infection is able to spread in the population. If it is below 1, the number of cases occurring in the population will gradually decrease to zero.



Source: Arroyo-Marioli F, Bullano F, Kucinskas S, Rondón-Moreno C (2021) Tracking R of COVID-19: A new real-time estimation using the Kalman filter. CC BY

Mortality

Mortality Risk

Q: If someone is infected with COVID-19,
how likely are they going to die from it?

Crude Mortality Rate

$$\text{Crude Mortality Rate} = \frac{\text{Number who have died from COVID-19}}{\text{Total population}}$$

e.g., if in a population of 10,000 people 200 people die from COVID-19, then the crude mortality rate = 2%.

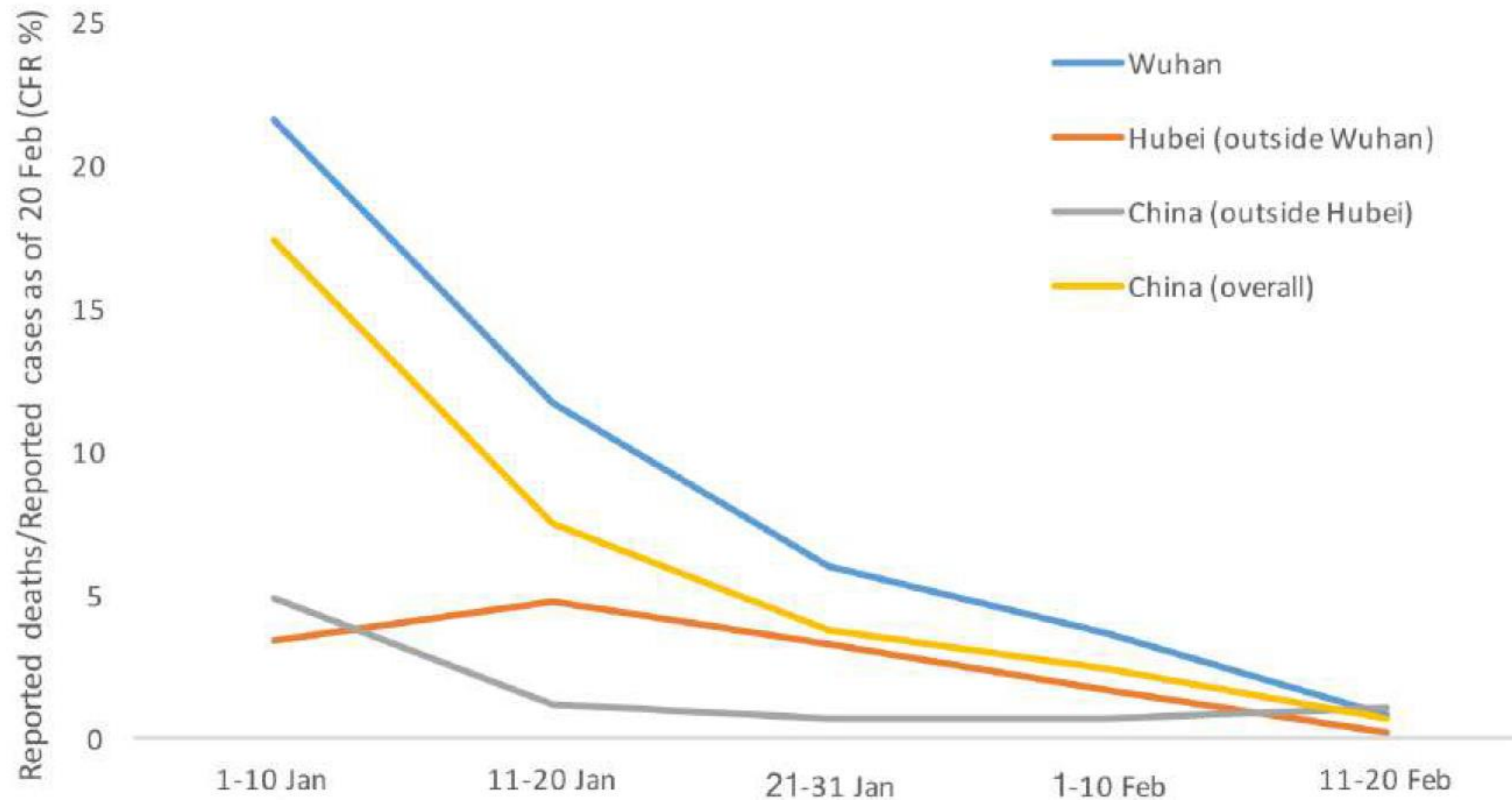
- NOT a measure of the true risk of death of someone who has COVID-19 since the denominator includes those without COVID-19.
- Often confused with the Case Fatality Rate (CFR).

Case Fatality Rate (CFR)

$$\text{CFR} = \frac{\text{Number of } \textit{confirmed} \text{ deaths from COVID-19}}{\text{Number of } \textit{confirmed} \text{ cases of COVID-19}}$$

- Again, cannot be interpreted as the risk of death for an infected person.
- *Confirmed* cases are those verified by a lab test result; but many cases go undiagnosed.
- May vary over time, between locations, by characteristics of infected population.

Example: CFR



World Health Organization (2020). Report of the WHO-China Joint Mission on Coronavirus Disease 2019 (COVID-19).

Infection Fatality Rate (IFR)

$$\text{IFR} = \frac{\text{Number of } \textit{actual} \text{ deaths from COVID-19}}{\text{Number of } \textit{actual} \text{ cases of COVID-19}}$$

e.g., if 10,000 people have COVID-19 and 200 die from it, then IFR = 2%.

- Number of *actual* cases is difficult to ascertain – relies upon testing coverage.
- A lot of literature describing how to estimate actual cases from samples.

Excess Mortality

Excess mortality is measured as the difference between the reported number of deaths in a period and an estimate of the expected deaths for that same period had the COVID-19 pandemic not occurred.

$$\text{Excess Deaths} = \text{Reported Deaths} - \text{Expected Deaths}$$

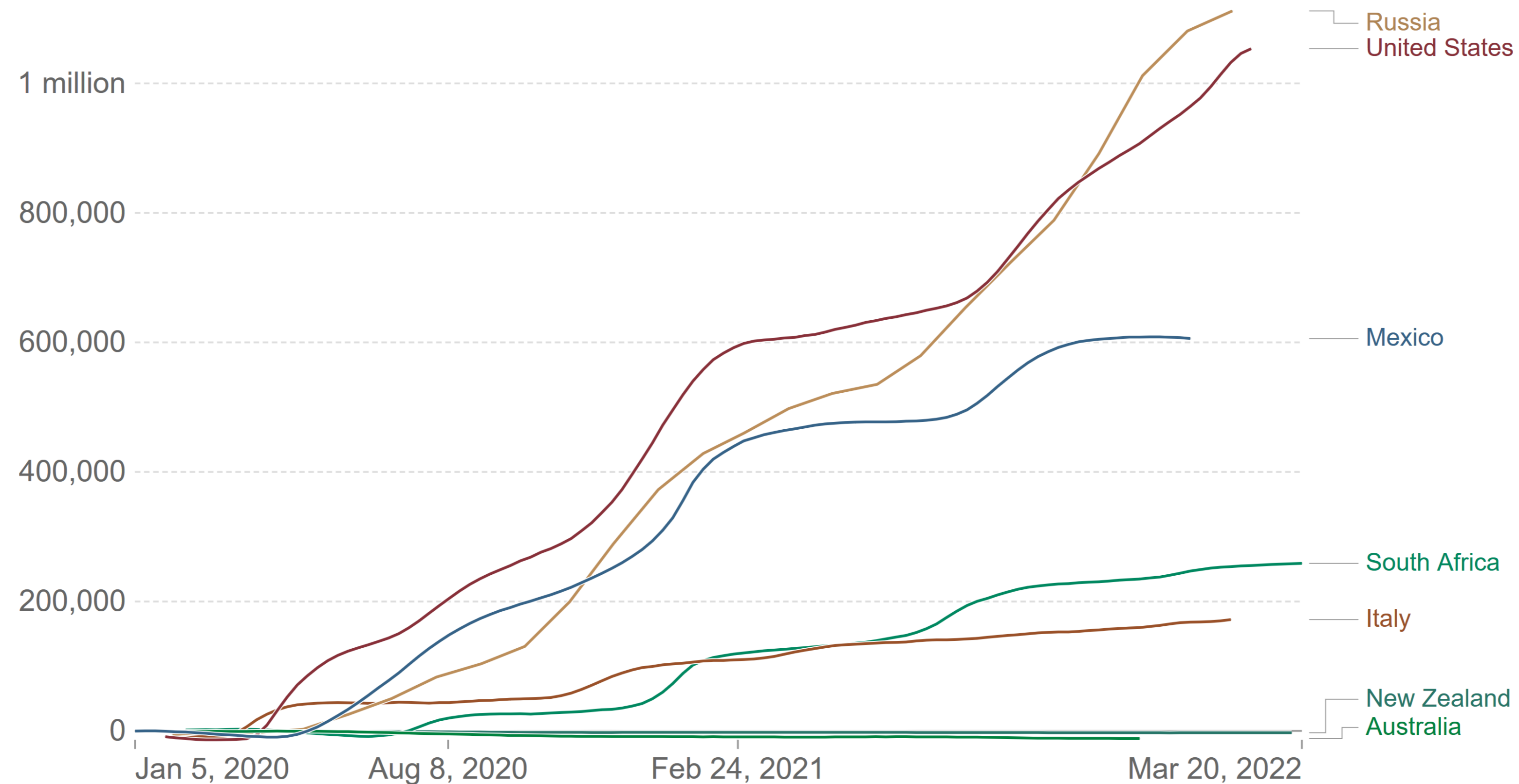
- Looks at All-Cause Mortality, not just COVID-19.
- Measures the *total* impact of the pandemic on deaths.
- Expected deaths is estimated using historical data.

Example: Excess Mortality

Excess mortality: Cumulative number of deaths from all causes compared to projection based on previous years



The cumulative difference between the reported number of deaths since 1 January 2020 and the projected number of deaths for the same period based on previous years. The reported number might not count all deaths that occurred due to incomplete coverage and delays in reporting.



Source: Human Mortality Database (2022), World Mortality Dataset (2022)

CC BY

Testing:

Without testing there is no data

Positive Rate

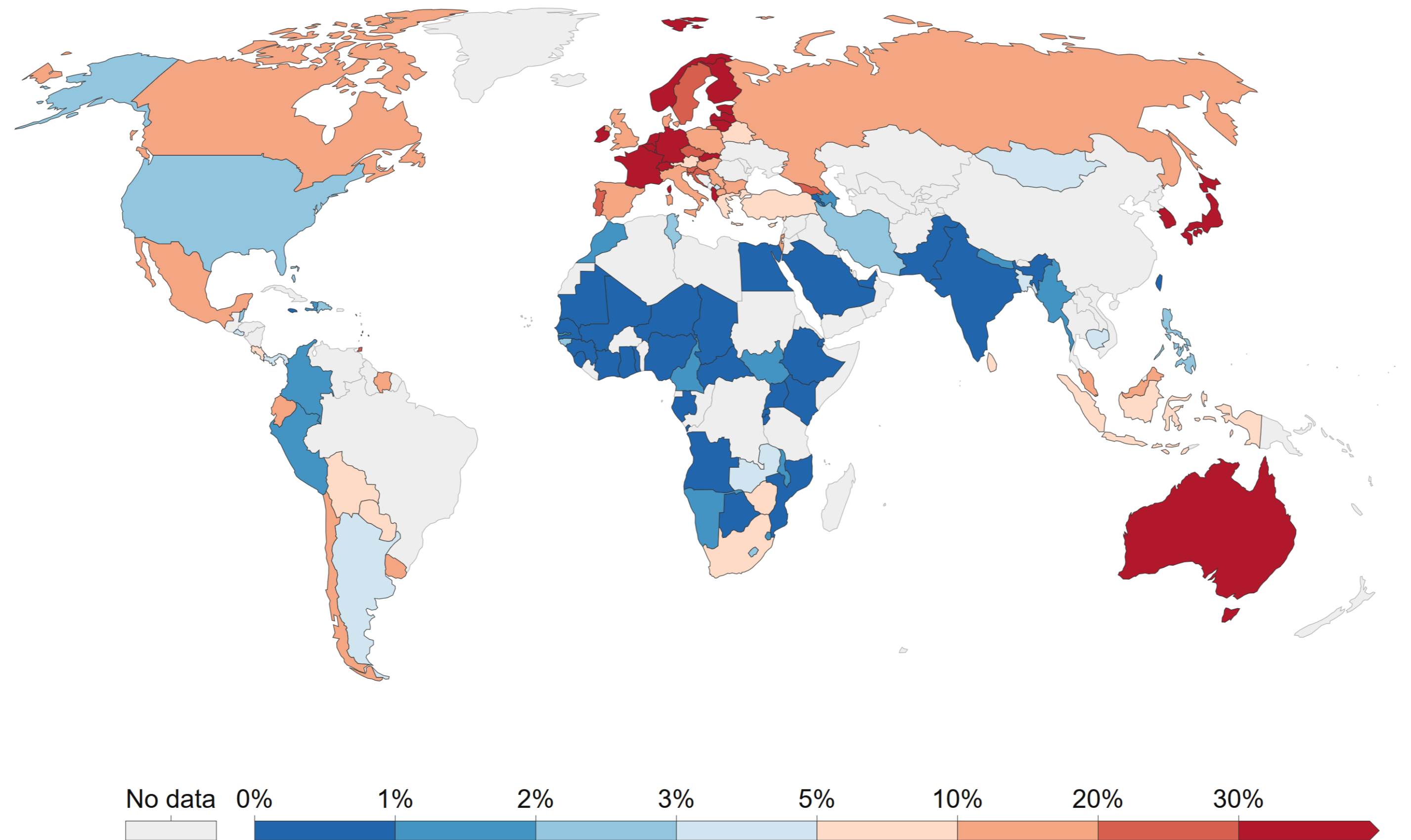
Positive Rate = Proportion of tests returning positive

- Reflects how adequately countries are testing relative to the size of the outbreak
 - Limited testing means many cases are missed → small Positive Rate
- A rising Positive Rate suggests COVID-19 is spreading faster than the growth in the observed confirmed cases.

Example: Positive Rate

The share of COVID-19 tests that are positive, Mar 28, 2022

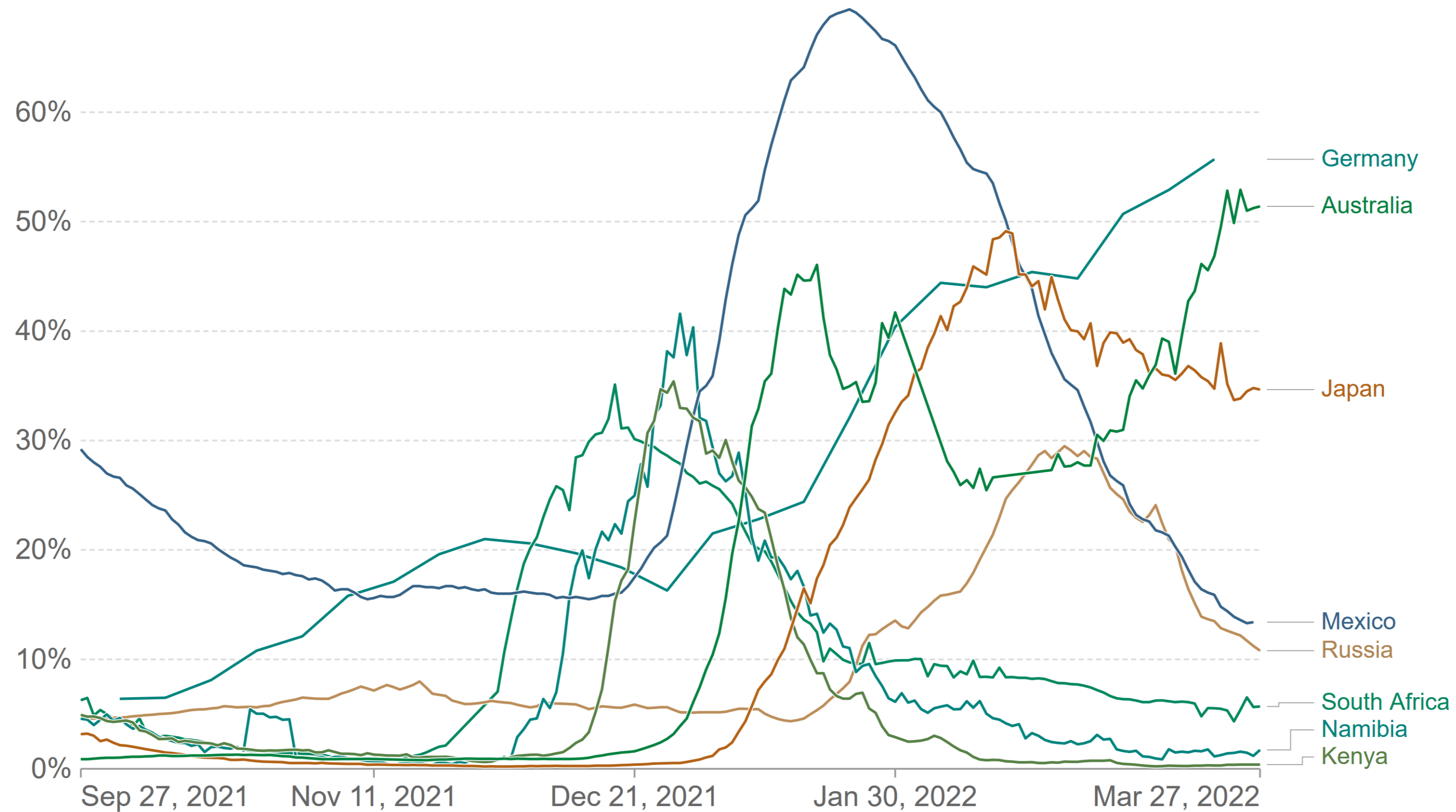
7-day rolling average. Comparisons across countries are affected by differences in testing policies and reporting methods.



Example: Positive Rate

The share of daily COVID-19 tests that are positive

7-day rolling average. The number of confirmed cases divided by the number of tests, expressed as a percentage. Comparisons across countries are affected by differences in testing policies and reporting methods.



Testing

Q: How “accurate” are COVID-19 tests?

Sensitivity & Specificity

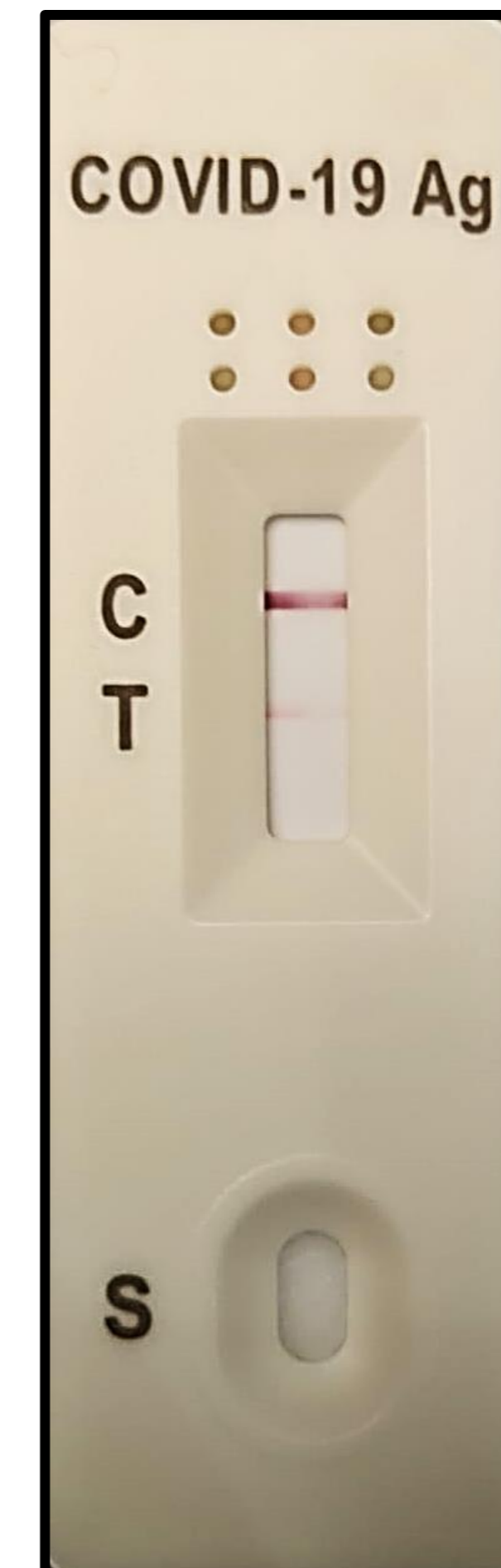
PERFORMANCE CHARACTERISTICS

Clinical performance

A clinical evaluation was conducted comparing the results obtained using the SARS-CoV-2 Antigen Rapid Test with RT-PCR test result. The clinical trial included 841 nasal swab specimens. The results demonstrated 99.4% specificity and 95.9% sensitivity with an overall accuracy of 98.0%.

	PCR confirmed sample number	Correct identified	Rate
Positive sample	341	327	95.9% (Sensitivity)
Negative sample	500	497	99.4% (Specificity)
Total	841	824	98.0% (Total Accuracy)

95.9% Sensitivity: In total 341 PCR confirmed positive samples: 327 PCR confirmed positive samples were correctly detected by SARS-CoV-2 Antigen Rapid Test. There are 14 false negative cases. 99.4% Specificity: In total 500 PCR confirmed negative samples: 497 PCR confirmed negative samples were correctly detected by SARS-CoV-2 Antigen Rapid Test. There are only 3 false positive cases. 98.0% Accuracy: In total 841 PCR confirmed samples: 824 PCR confirmed samples were correctly detected by SARS-CoV-2 Antigen Rapid Test.



Sensitivity

$$\text{Sensitivity} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Negatives}}$$

Correctly returning a *positive* result for someone who *does* have COVID-19.

Reduced when someone is incorrectly identified as *NOT having* COVID-19 when they actually *do* (False Negative).

Specificity

$$\text{Specificity} = \frac{\text{True Negatives}}{\text{True Negatives} + \text{False Positives}}$$

Correctly returning a *negative* result for someone who *does not* have COVID-19.

Reduced when someone is incorrectly identified as *having* COVID-19 when they actually *do not* (False Positive).

Example

Actual Disease Status

Infected

Not Infected

Test Result

Positive

True Positive
327

False Positive
3

Negative

False Negative
14

True Negative
497

$$\text{Sensitivity} = \frac{327}{327 + 14} = 95.9\%$$

$$\text{Specificity} = \frac{497}{497 + 3} = 99.4\%$$

$$\text{Accuracy} = \frac{327 + 497}{841} = 98.0\%$$

Positive Predictive Value (PPV)

$$PPV = P(\text{Have COVID-19} \mid \text{Tested Positive}) = \frac{TP}{TP + FP}$$

- Answers the question: If test results are positive in a patient, what is the probability that they have COVID-19?
- More influenced by Specificity than Sensitivity of test if cases are few (i.e., low prevalence) since most of the population is negative for the disease.

$$PPV = \frac{327}{327 + 3} = 99.1\%$$

Inferential Statistics

Inferential Statistics

- By statistical inference we mean going beyond just describing the state or distribution of our sample data and rather make conclusions about the target population.
- Any conclusions are therefore subject to some uncertainty, not least due to the variability in our data sampling.
- In Classical statistics, one framework for making this inductive leap from sample to population is *Null Hypothesis Significance Testing* (NHST).

Hypothesis Testing

From our research question we formulate two hypotheses, typically:

- H_0 : A treatment or intervention has zero effect (Null).
- H_1 : A treatment or intervention has a non-zero effect (Alternative).

We then proceed to collect data and depending on how much our sample appears compatible with the assumption that H_0 is true, we may reject or fail to reject H_0 in favour of H_1 .

p-values

A p-value is the probability, assuming that the **null hypothesis is true**, of observing a test statistic that is the same or more extreme than that observed in the collected data.

- It is NOT the probability that the null hypothesis is true!
- Consider it a measure of your surprise at seeing your sample when all along you've assumed that H_0 was true.
 - The smaller the p-value the less compatible the data seems with H_0 and the more plausible to reject it.
- But how small is small?

Type I and Type II Errors

Under the NHST framework, it is possible to make an error with respect to deciding between H_0 and H_1 :

- **Type I Error** (“False Positive”)
 - incorrectly rejecting H_0 , i.e., deciding that there **is a real effect** when in reality **there is none**.
- **Type II Error** (“False Negative”)
 - incorrectly failing to reject H_0 , i.e., deciding that **no real effect** was observed when in reality **there is one**.

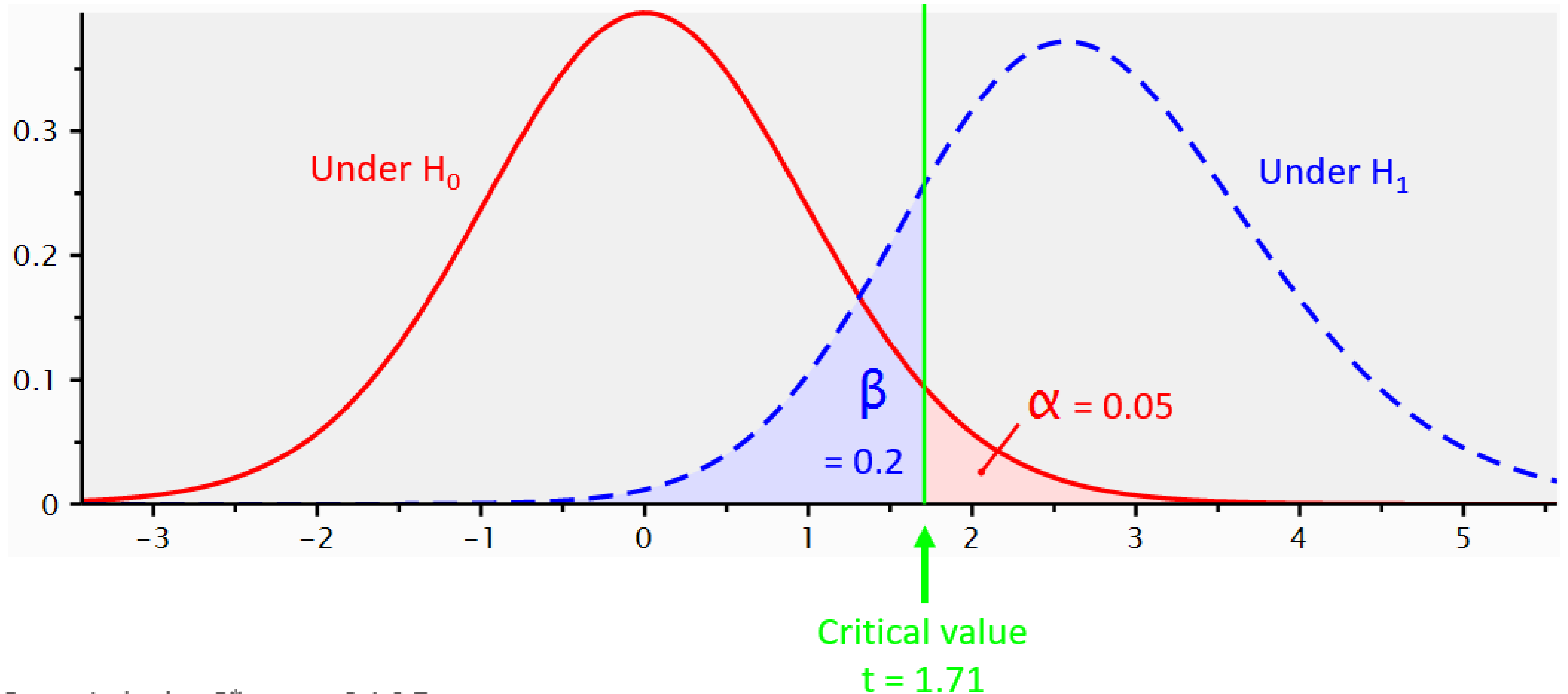
Significance Level $\alpha = 0.05$

- We don't know for a particular statistical test whether we've made a Type I error or Type II error or neither.
- But we can control our *long-run* Type I Error Rate by making decisions based on a threshold α ("alpha"):
 - p-value below $\alpha \rightarrow$ decide to reject H_0 ,
 - p-value above $\alpha \rightarrow$ decide in favour of H_0 .
- This threshold $P(\text{Type I Error}) = \alpha$ is by convention often set at 5%, i.e., if we were to repeat our experiment 100 times we'd expect to erroneously reject H_0 only 5 times.
- What about Type II errors?

Statistical Power ($1 - \beta$)

- The probability of making a Type II Error is denoted β (“beta”).
- The complement of β , known as **statistical power**, is the probability of correctly rejecting a false H_0 ,
e.g., correctly concluding that a treatment **has an effect** when in reality it does.
- Underpowered studies, assuming that H_0 is false, may overlook real treatment effects that are very small.
- This leads into Power Analysis and sample size calculation.

Example: Independent sample t-test



Effect Size

- p-values provide no indication of the magnitude of the effect of a treatment, whether “statistically significant” or not.
- It is common to report “standardised effect sizes” which have been scaled so that they are unitless and more easily compared between studies.
 - Pearson correlation, r
 - Cohen’s d
 - Odds ratio
 - Relative Risk
- Cohen (1988) suggested some conventions for what constitutes small, medium, and large effect sizes.

Example: Cohen's d

$$d = \frac{\text{mean(Treatment Group)} - \text{mean(Comparator Group)}}{\text{SD(Pooled Sample)}}$$

- Standardised mean differences, e.g., $d = 1$ indicates two groups' means differ by 1 standard deviation.

Effect Size	Cohen's d
"Small"	0.20
"Medium"	0.50
"Large"	0.80

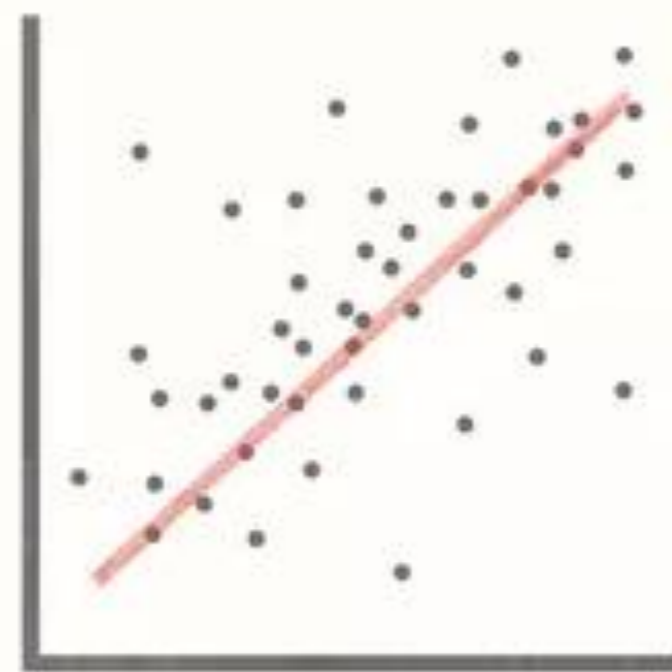
Sample Size

- For a statistical test the following factors are inter-related:
 - Level of significance, $\alpha = P(\text{Type I error})$
 - Statistical power, $(1 - \beta) = 1 - P(\text{Type II error})$
 - Sample size, n
 - Minimal effect size of interest
- Knowing three of the above lets you compute the fourth.

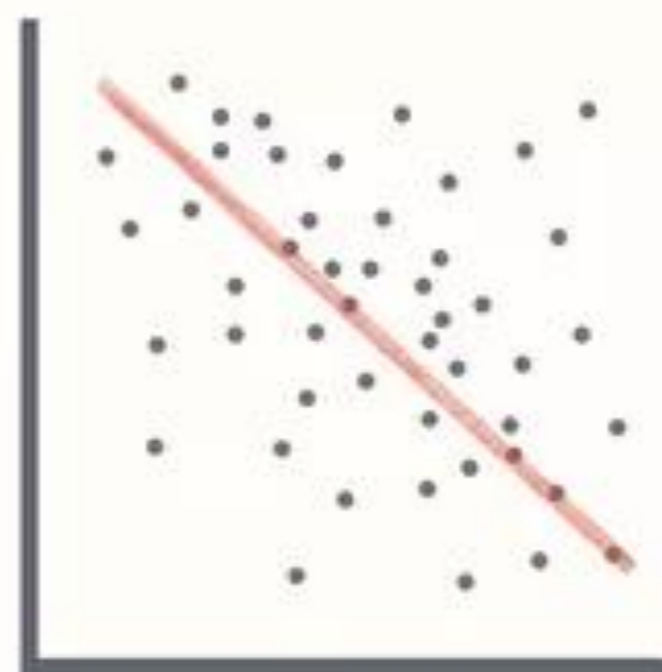
Correlation

$$r = \frac{\text{Covariance}(X, Y)}{\text{SD}(X) \times \text{SD}(Y)}$$

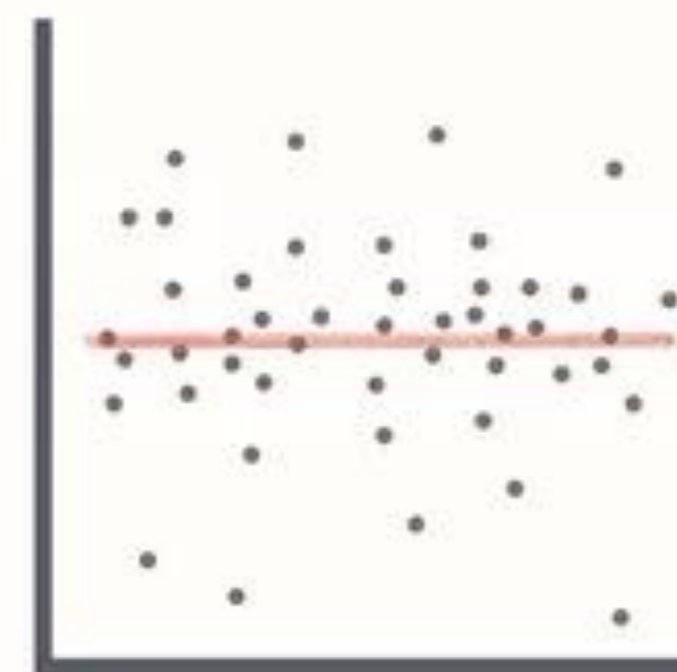
- Ranges between -1 and +1.
- Measure of the strength of *linear association* between two variables



Positive Correlation

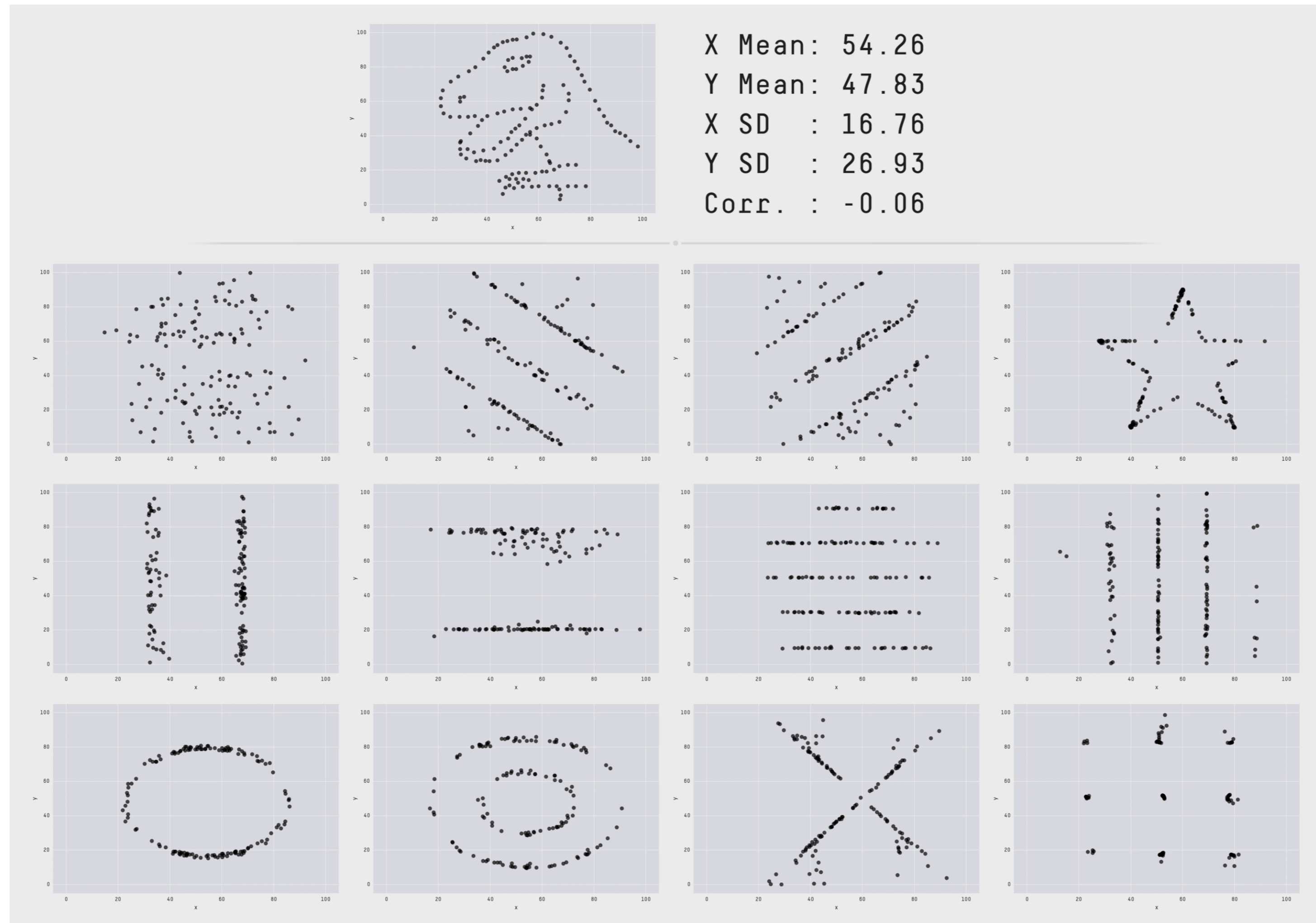


Negative Correlation



No Correlation

Example: The Datasaurus Dozen



Justin Matejka and George Fitzmaurice. 2017. Same Stats, Different Graphs: Generating Datasets with Varied Appearance and Identical Statistics through Simulated Annealing. DOI: <https://doi.org/10.1145/3025453.3025912>

What is CTEP-WA?

- Formerly called Clinical Trials and Data Management Centre (CTDMC)
- Expertise in clinical trial study design, clinical trial conduct, data management, data linkage, analytical techniques for clinical trial datasets, bio-repository techniques, and clinical registry datasets.
- Biostatistical consultation service to clients conducting clinical research.
- CTEP-WA also offers an Auditing and Monitoring Service for Investigator-Initiated clinical trials.



Thank you

[@ShihChingFu](#)

shihching.fu@curtin.edu.au

