Gaussian process regression for identifying variables and transients in ThunderKAT

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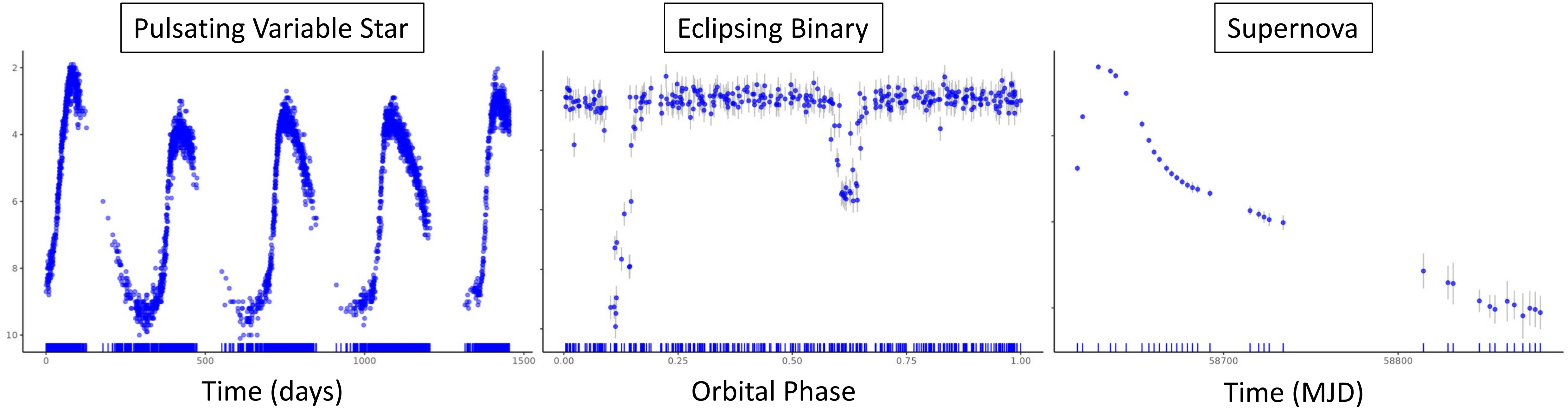
Light curves as stochastic processes

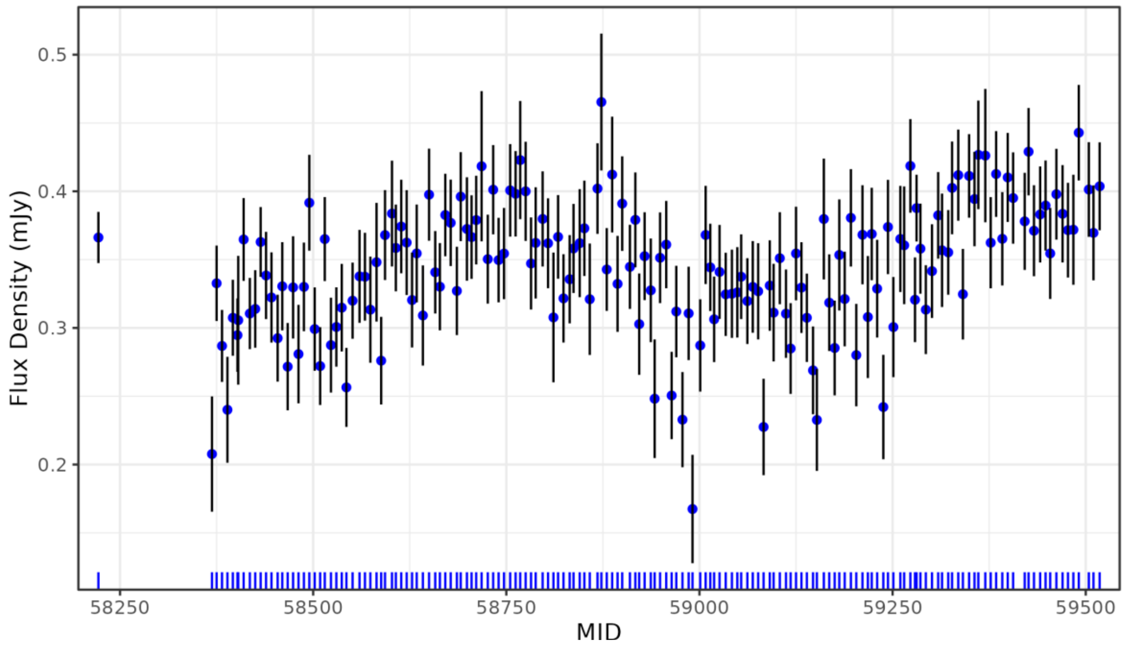
Light curves are time series describing the brightness of a source over time.

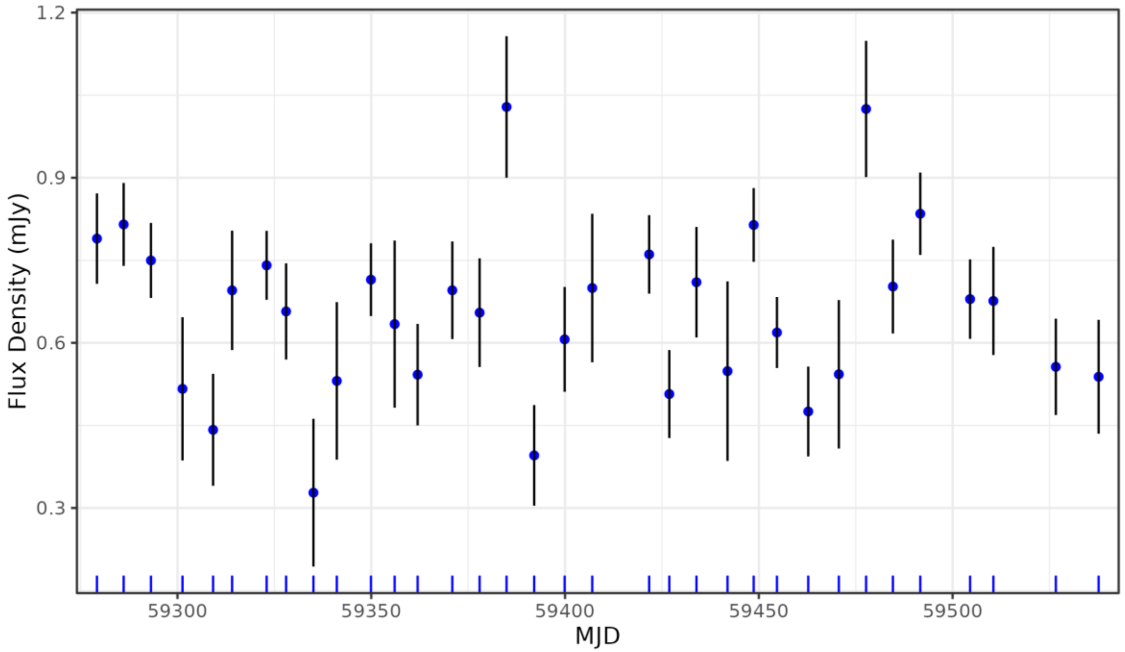
- Consider them a *stochastic process.*
- Random variables indexed in time with *autocovariance* structure.

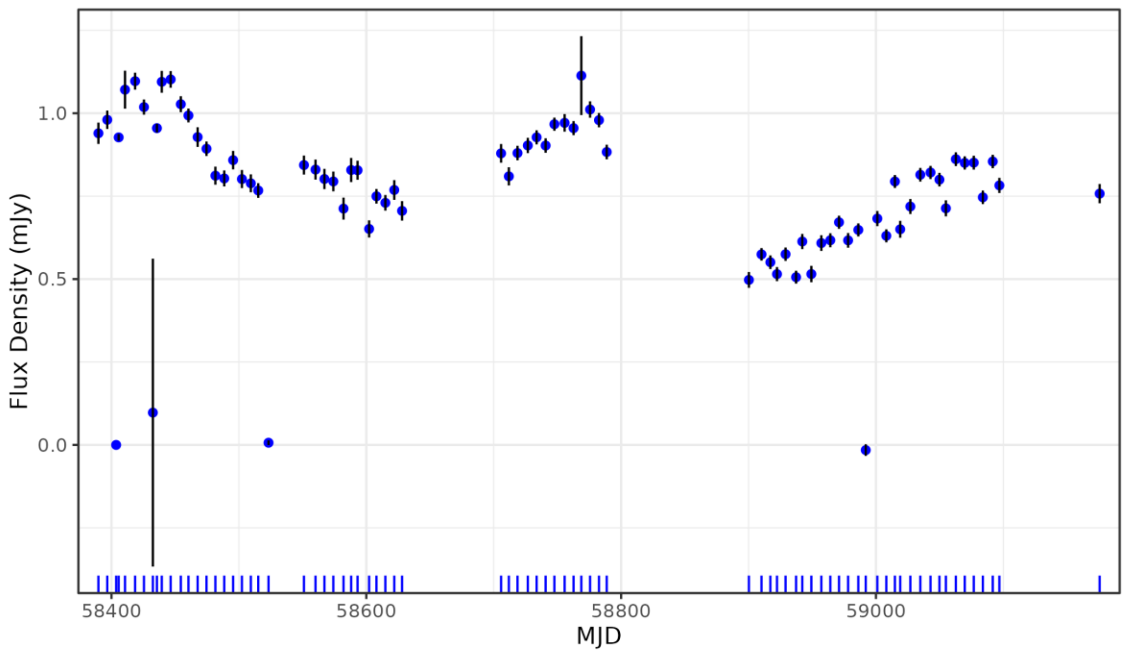


Data is heterogeneous

- Different cadences
- Sparse observations
- Irregular sampling
- Varying noise levels



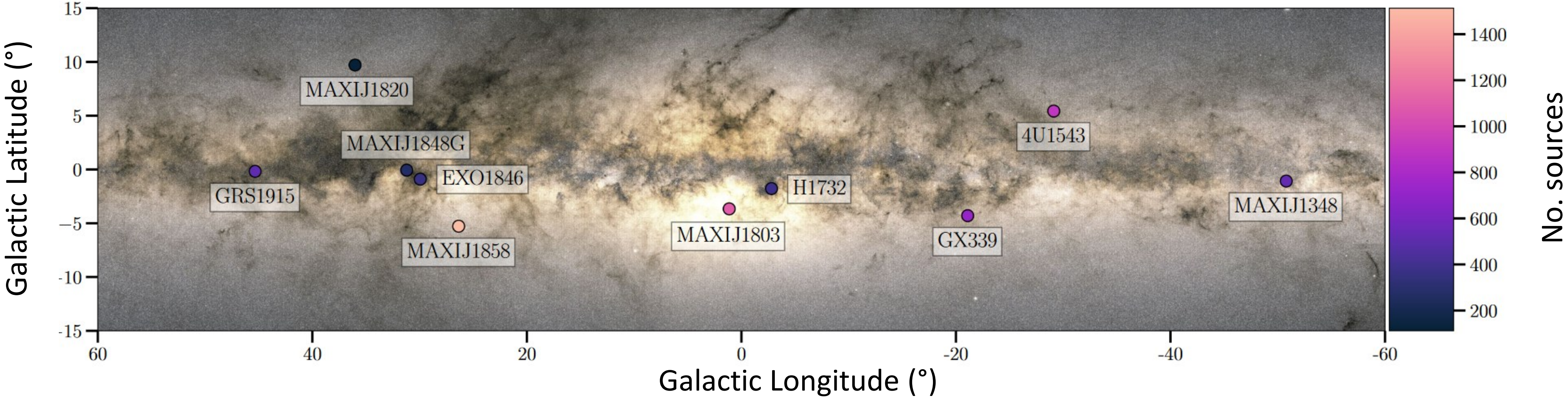




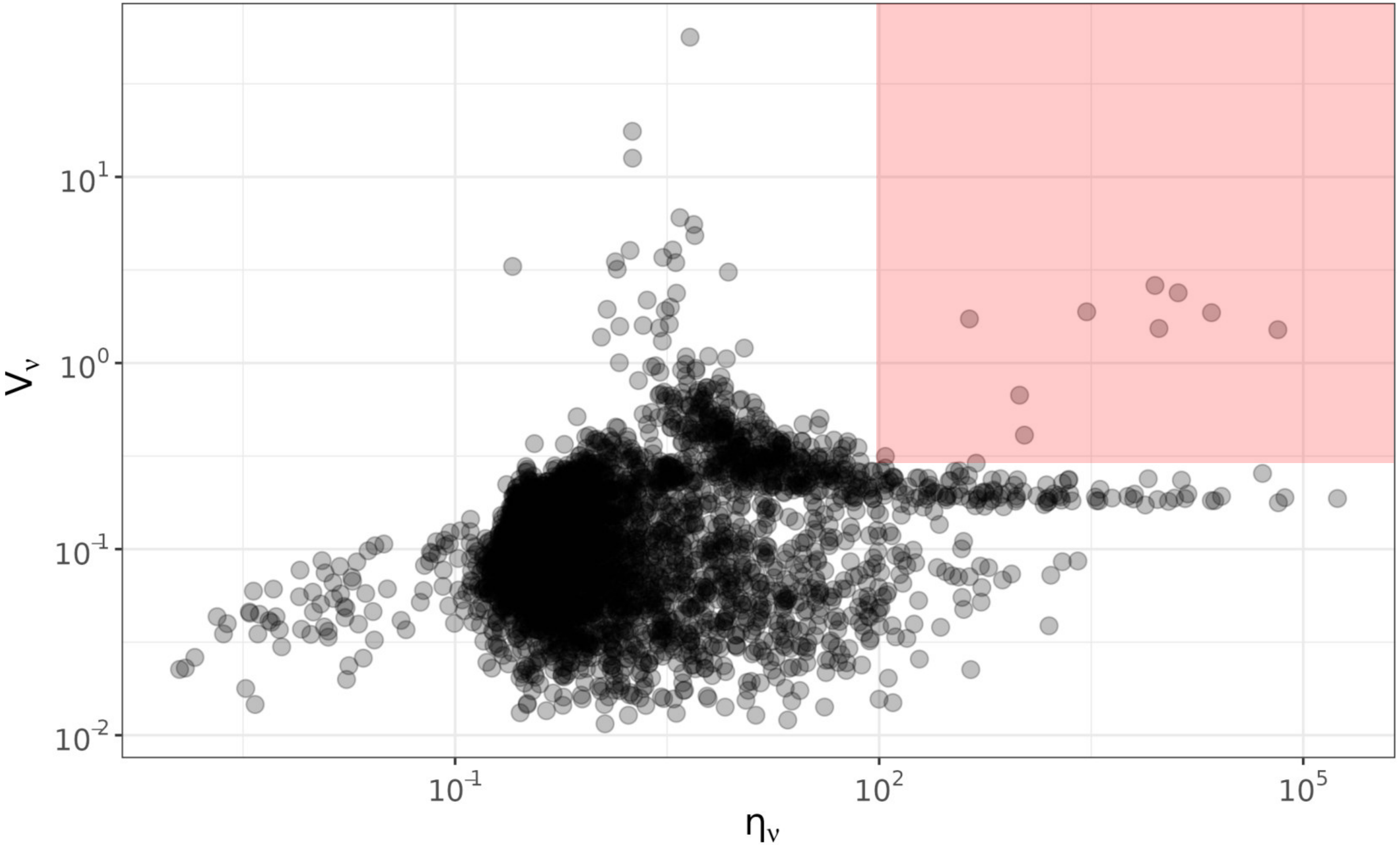
Dataset

- The HUNt for Dynamic and Explosive Radio transients with MeerKAT
- Field of view of ≈ 1 square degree
- 6,394 radio light curves over 10 fields
- Flux density measurements + standard errors

MeerKAT Radio Telescope (Credit: SARAO)



Variability Statistics: η_ν and V_ν



(Data courtesy of Andersson et al., 2023)

$$\eta_\nu = \frac{1}{N}\sum\left(\frac{\text{Obs.} - \text{Wt. Mean}}{\text{Std. Error}}\right)^2$$

$$\sim \chi^2_{N-1}$$

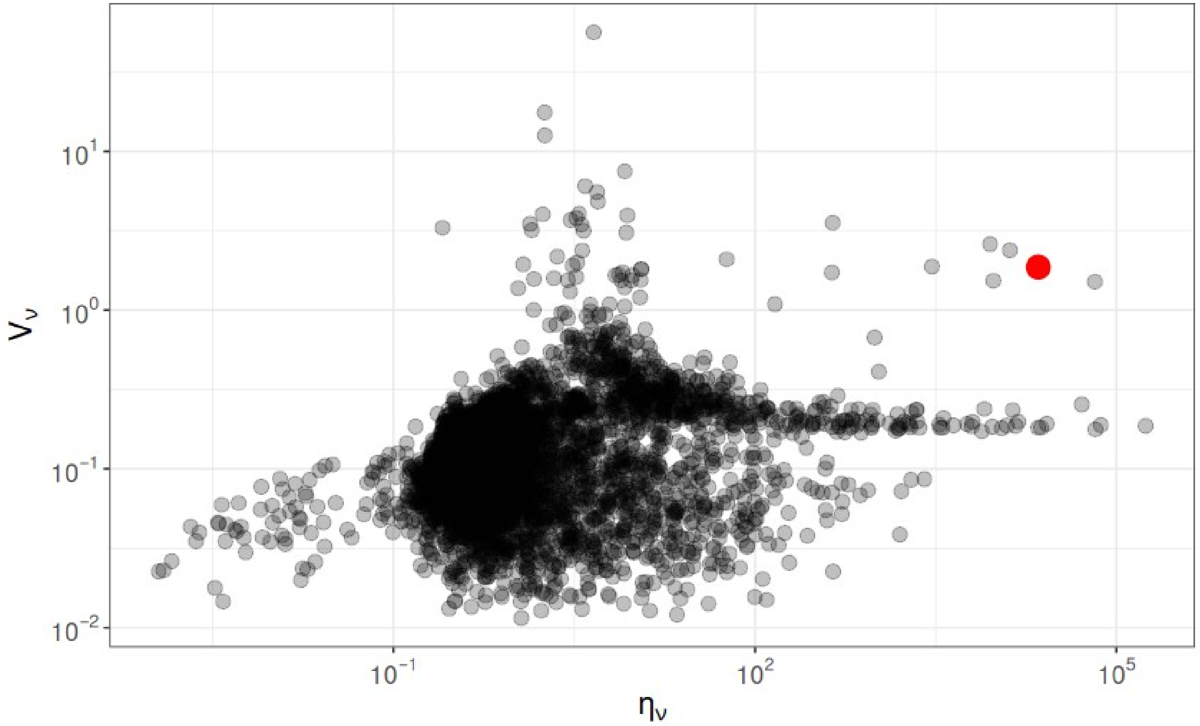
$$V_\nu = \frac{\text{Standard Deviation}}{\text{Mean}}$$

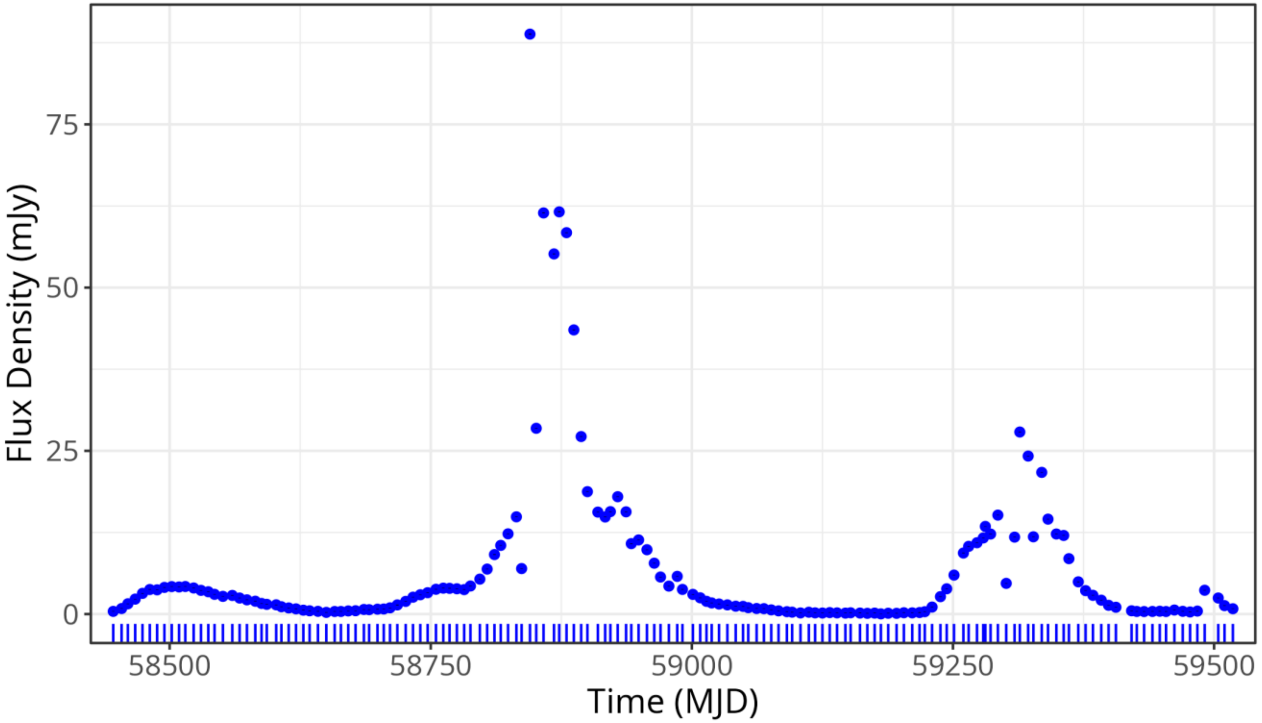
As $\eta_\nu \to \infty$ and $V_\nu \to \infty$

Source is likely transient

GX 339-4 in (η_ν, V_ν)-space

- Galactic LMXB and black hole candidate that flares





$$(\eta_\nu = 22427.6, V_\nu = 1.86)$$

Characterising Light Curves

Oversimplified

- Fewer parameters
- Scales easily
- High information loss

Overspecified

- Many parameters
- High discriminatory power
- Risks overfitting

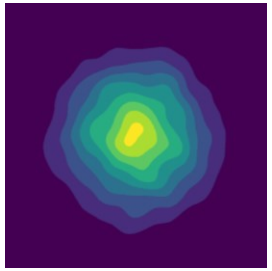
Model light curves as a Gaussian Process (GP)

Multivariate Normal $\mathbf{Y} \sim \mathrm{MVN}(\mathbf{0}, \boldsymbol{\Sigma}_{n\times n})$

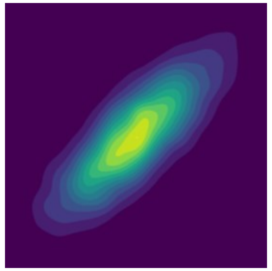
$\boldsymbol{Y}$ is a vector of n Gaussian random variables.

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix} = \boldsymbol{Y} \sim MVN(\boldsymbol{\mu}, \boldsymbol{\Sigma}_{n\times n}), \qquad \boldsymbol{\Sigma}_{n\times n} = \begin{bmatrix} \Sigma_{11} & \cdots & \Sigma_{1n} \\ \vdots & \ddots & \vdots \\ \Sigma_{n1} & \cdots & \Sigma_{nn} \end{bmatrix}$$

where $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_n)$ and $\boldsymbol{\Sigma}$ is a $n \times n$ covariance matrix.



$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\Sigma = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$$

- Symmetric, positive semi-definite matrix.
- Linear combinations of covariance matrices are also valid covariance matrices.

Gaussian Processes (GPs)

Extend multivariate Gaussian to 'infinite' dimensions.

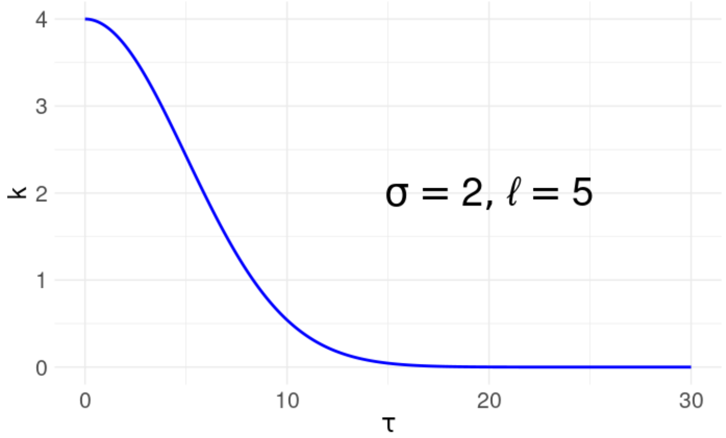
- Mean function, $\mu(t)$
- Covariance or **kernel function**, $\boldsymbol{\kappa}(\boldsymbol{t}, \boldsymbol{t})$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \end{bmatrix} = \boldsymbol{Y} \sim GP(\mu(t), \boldsymbol{\Sigma})$$

where $\boldsymbol{\mu} = \mu(t_i)$ and $\Sigma_{ij} = \boldsymbol{\kappa}(\boldsymbol{t_i}, \boldsymbol{t_j})$, for $i, j = 1, 2, \ldots$

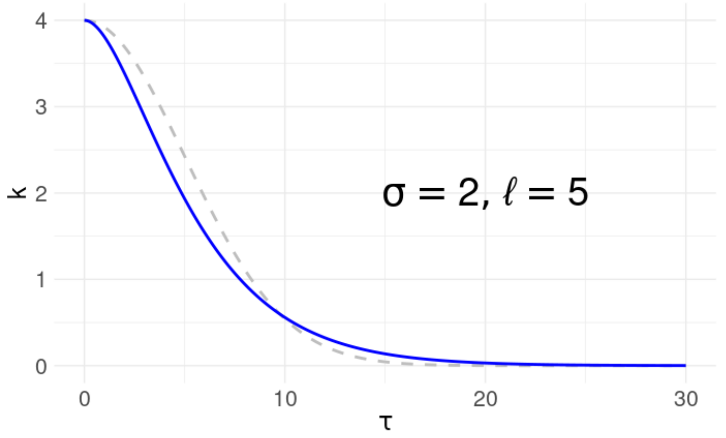
Rather than specifying a fixed covariance matrix with fixed dimensions, compute covariances using the kernel function.

$$\tau = |t_r - t_c|;\ \sigma, \ell, T > 0$$



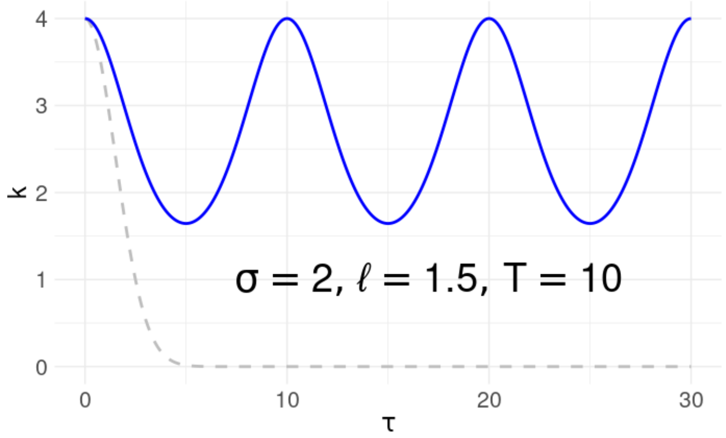
$$\kappa(\tau; \sigma, \ell) = \sigma^2 \exp\left\{-\frac{1}{2}\left(\frac{\tau}{\ell}\right)^2\right\}$$

Squared Exponential



$$\kappa(\tau; \sigma, \ell) = \sigma^2 \left(1 + \sqrt{3}\frac{\tau}{\ell}\right) \exp\left\{-\sqrt{3}\frac{\tau}{\ell}\right\}$$

Matern 3/2

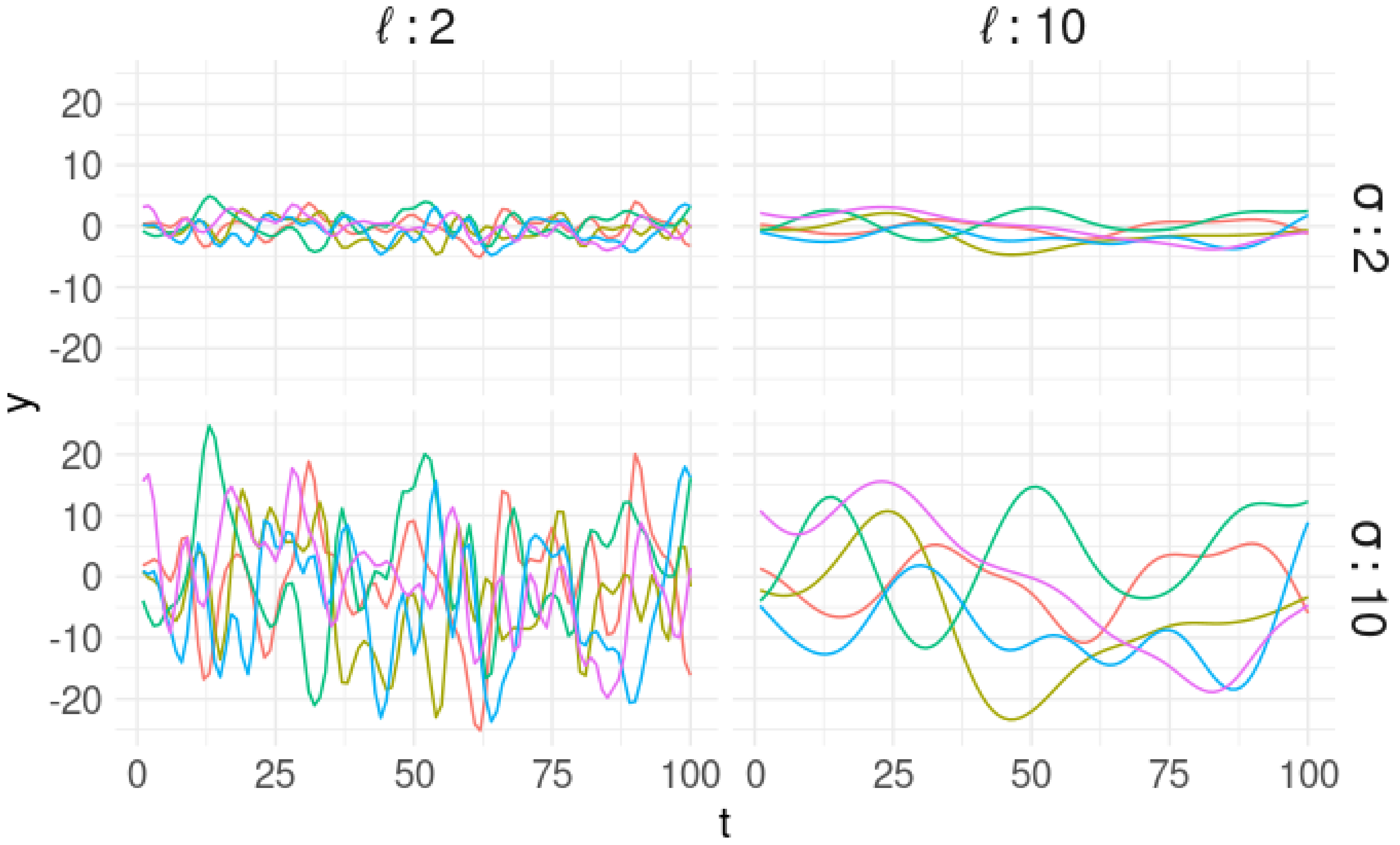


$$\kappa(\tau; \sigma, \ell, T) = \sigma^2 \exp\left\{-\frac{2}{\ell^2}\sin^2\left(\pi\frac{\tau}{T}\right)\right\}$$

Periodic

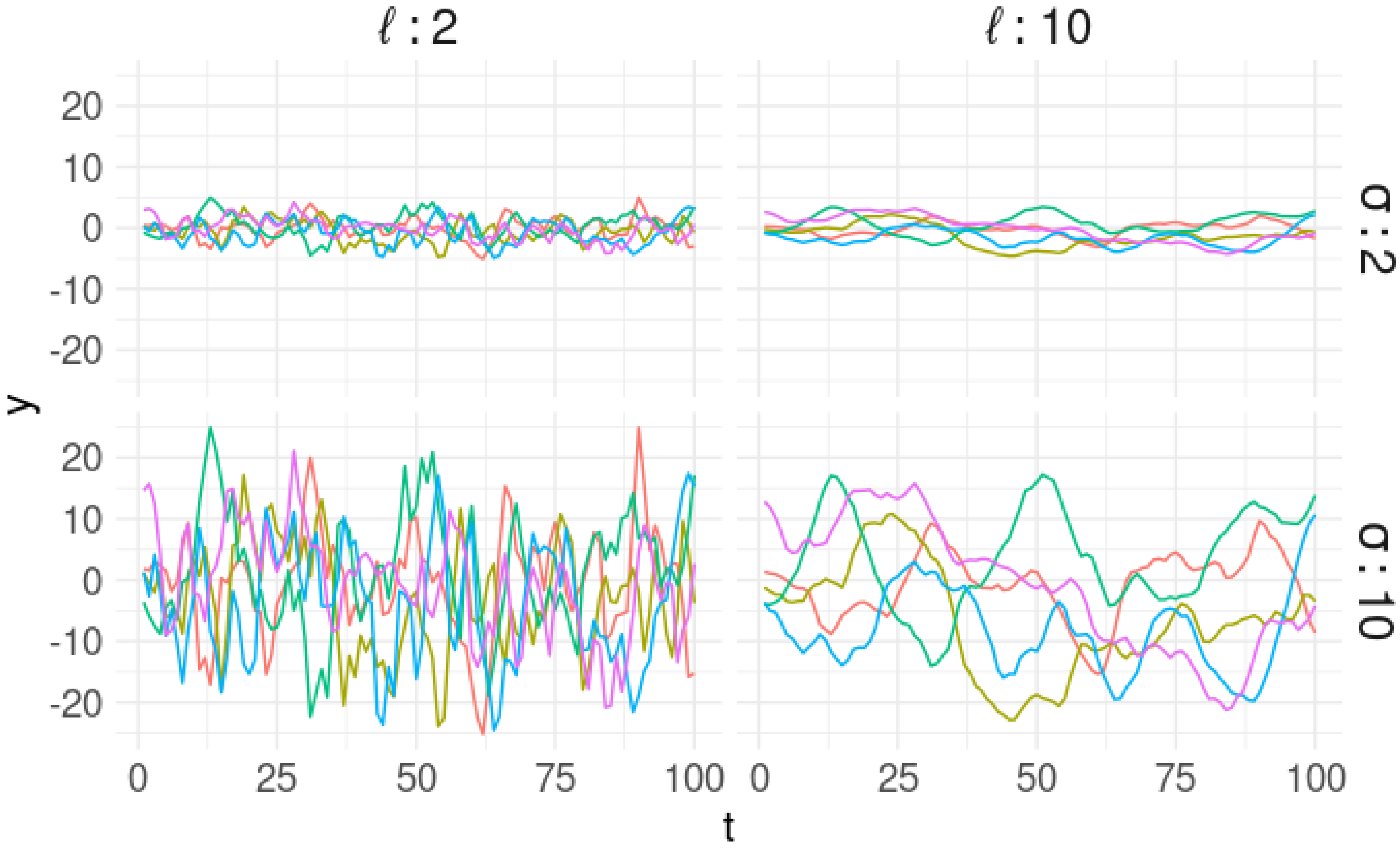
Squared Exponential Kernel

$$\kappa(\tau;\sigma,\ell)=\sigma^2\exp\left\{-\frac{1}{2}\left(\frac{\tau}{\ell}\right)^2\right\}$$



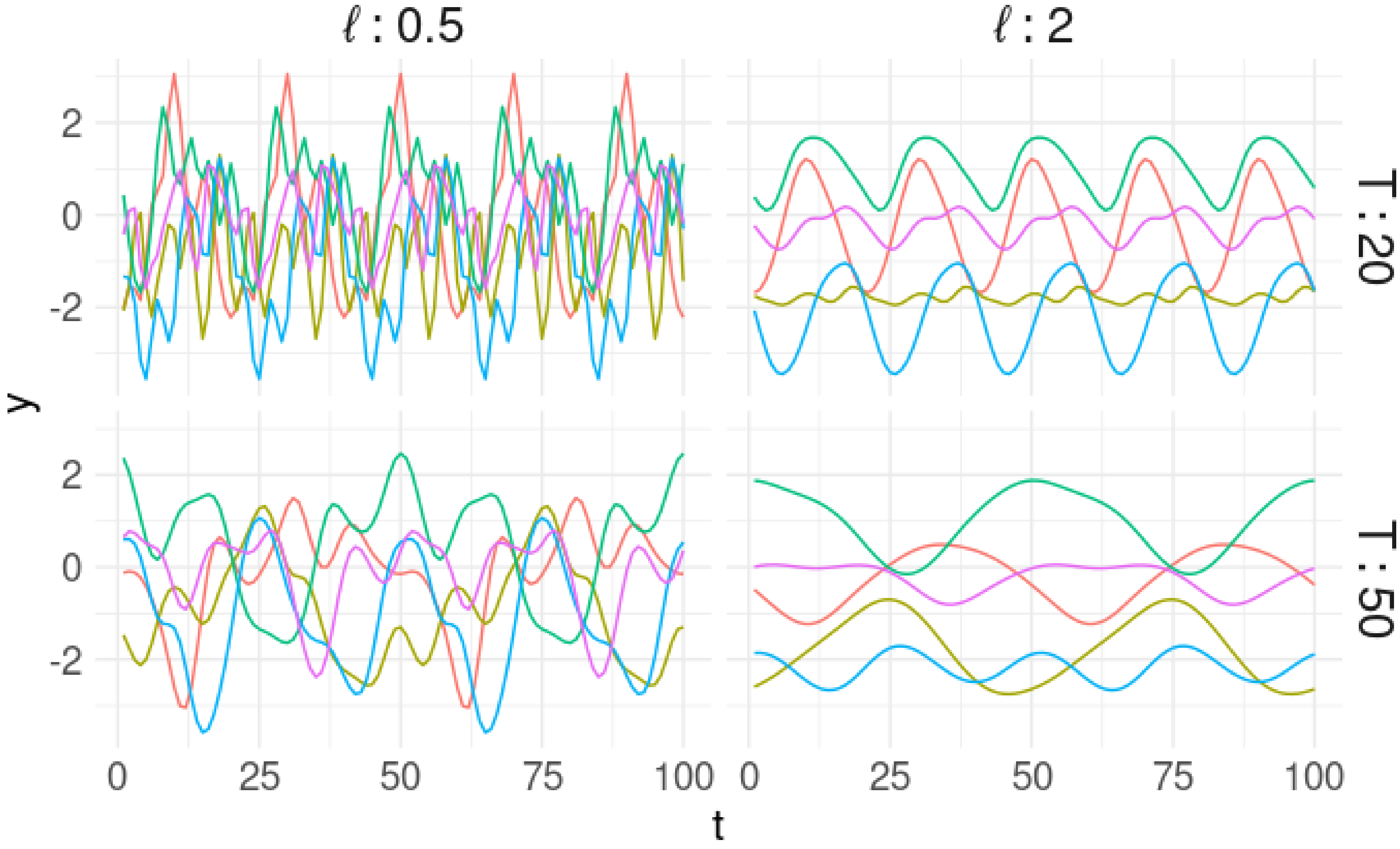
Matern 3/2 Kernel

$$\kappa(\tau; \sigma, \ell) = \sigma^2 \left(1 + \sqrt{3}\frac{\tau}{\ell}\right) \exp\left\{-\sqrt{3}\frac{\tau}{\ell}\right\}$$



Periodic Kernel

$$\kappa(\tau; \sigma, \ell) = \sigma^2 \exp\left\{-\frac{2}{\ell^2}\sin^2\left(\pi\frac{\tau}{T}\right)\right\}$$



Bayesian Hierarchical Model

Data Model $\mathbf{Y} \sim \mathcal{N}_N(\mathbf{f}, \hat{e}^2)$ Observed Error

Process Model $\mathbf{f} \sim \mathcal{GP}(\mathbf{0}, \mathbf{\Sigma}_{N\times N})$

$$r, c = 1, \dots, N.$$

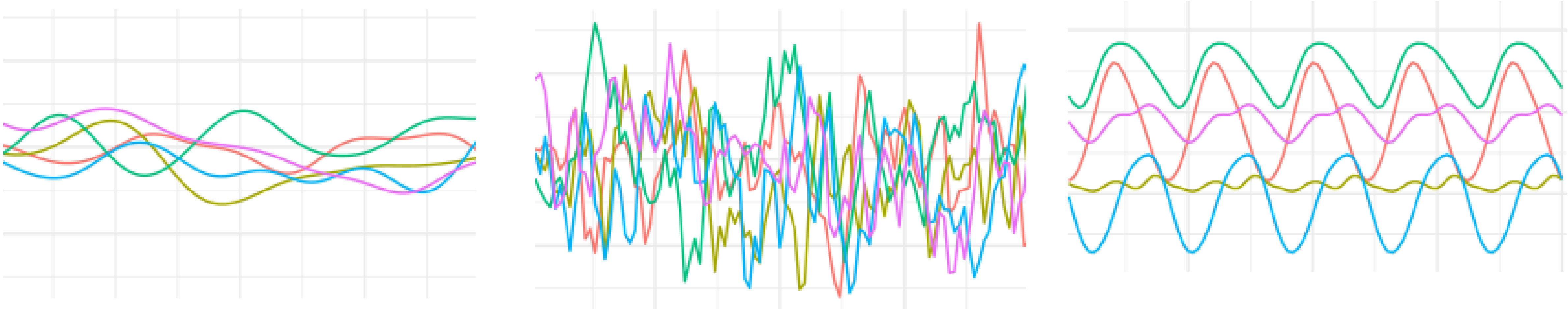
$$\boldsymbol{\theta} = \{\sigma_{SE}, \ell_{SE}, \sigma_{M32}, \ell_{M32}, \sigma_P, \ell_P, T\}$$

$$[\mathbf{\Sigma}]_{rc} = \kappa(t_r, t_c | \boldsymbol{\theta})$$

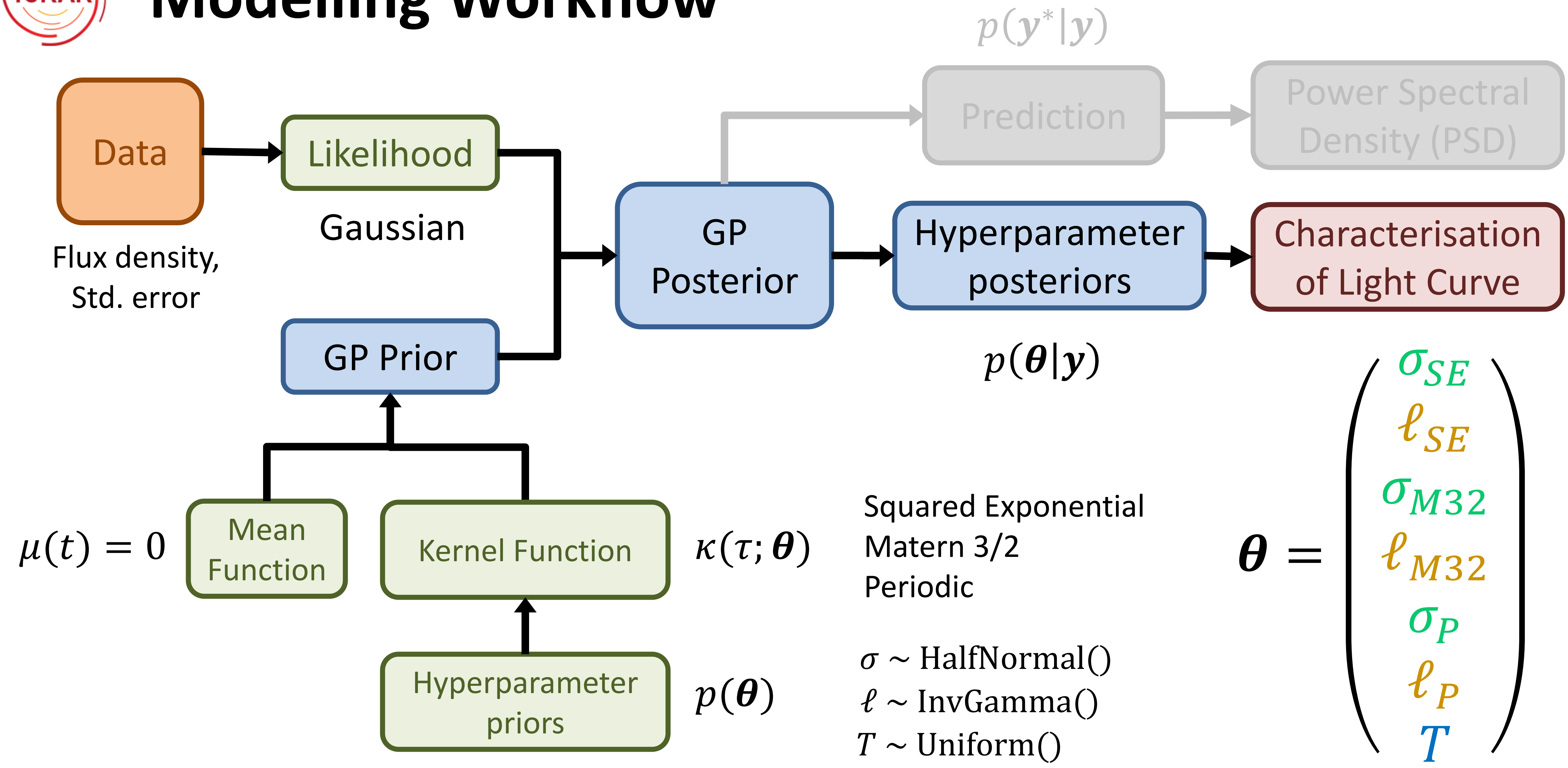
$$= \kappa_1(\tau; \sigma_{SE}, \ell_{SE}) + \kappa_2(\tau; \sigma_{M32}, \ell_{M32}) + \kappa_3(\tau; \sigma_P, \ell_P, T)$$

Squared Exponential | Matern 3/2 | Periodic

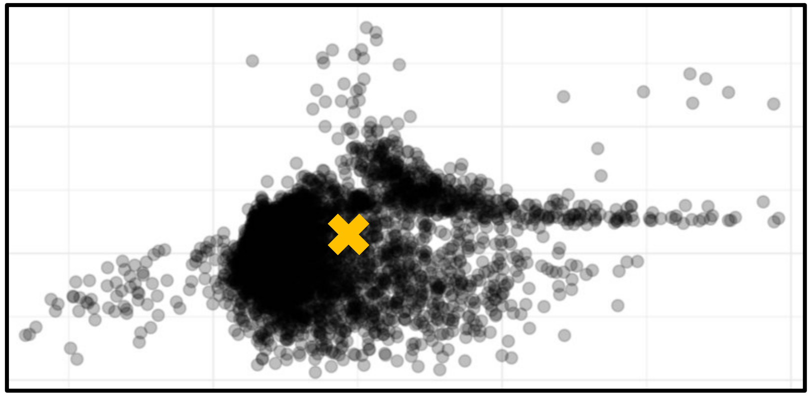
Covariance Kernel



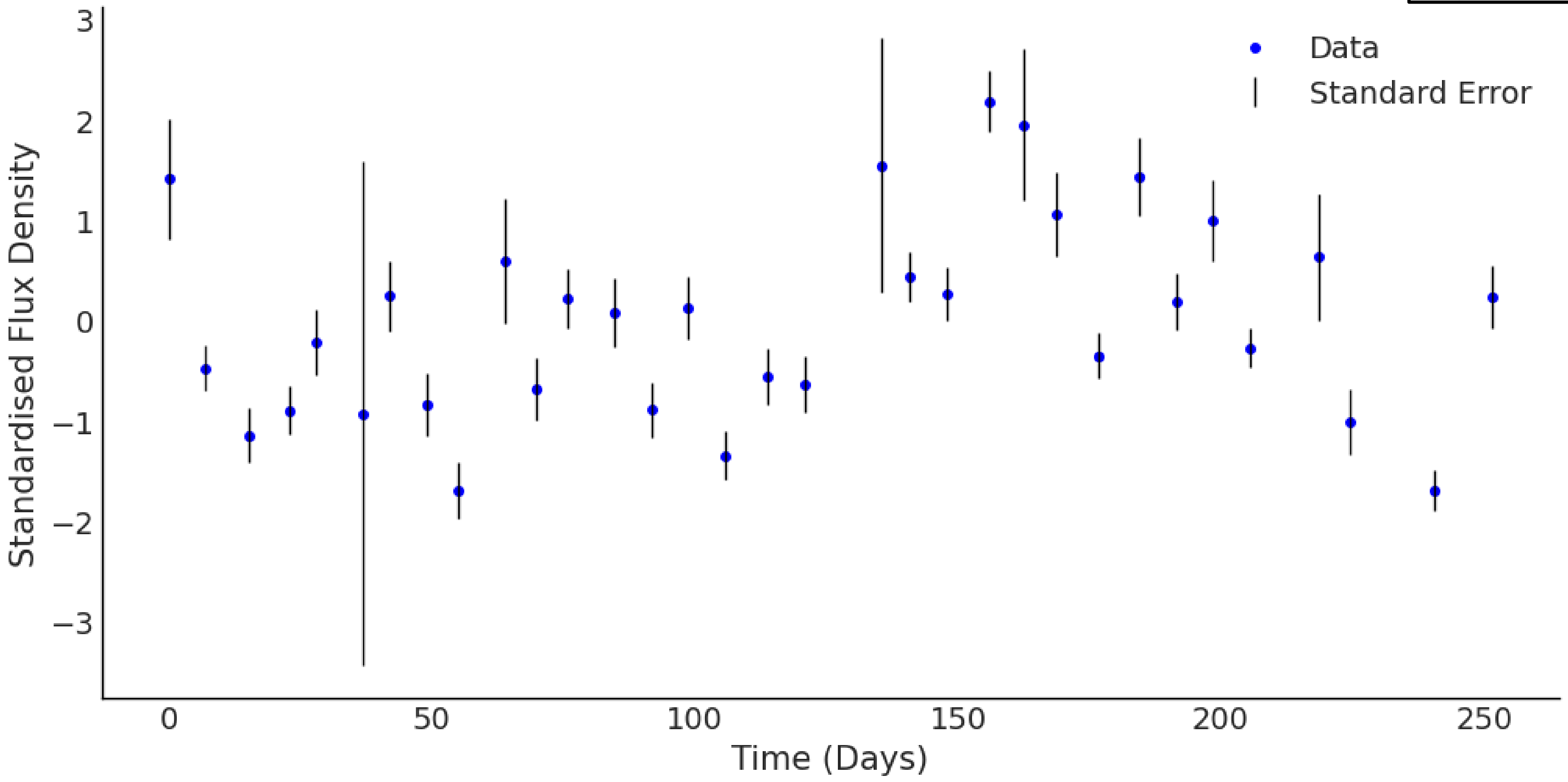
Modelling Workflow



Example



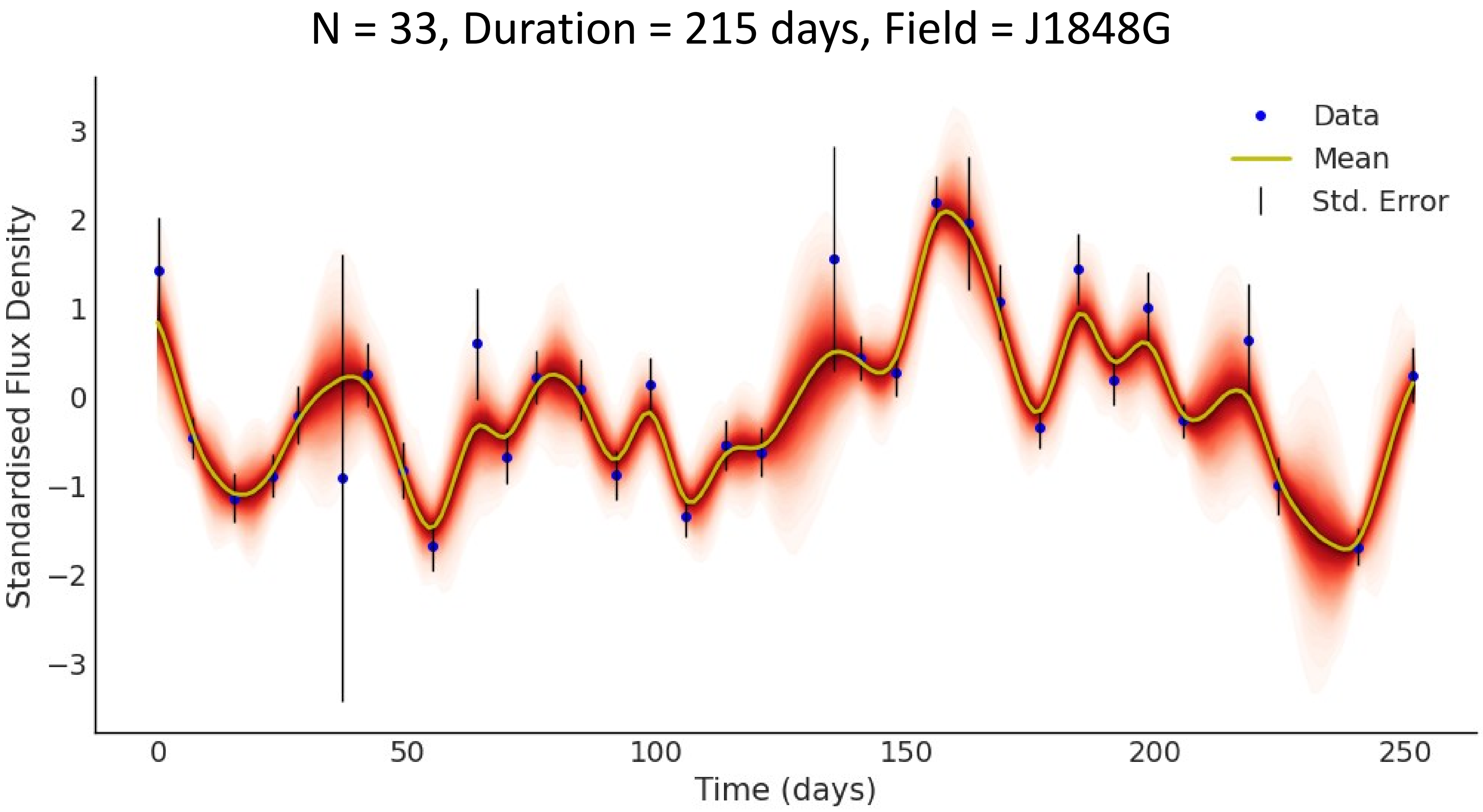
N = 33, Duration = 215 days, Field = J1848G



$\eta_\nu = 2.91$

$V_\nu = 0.12$

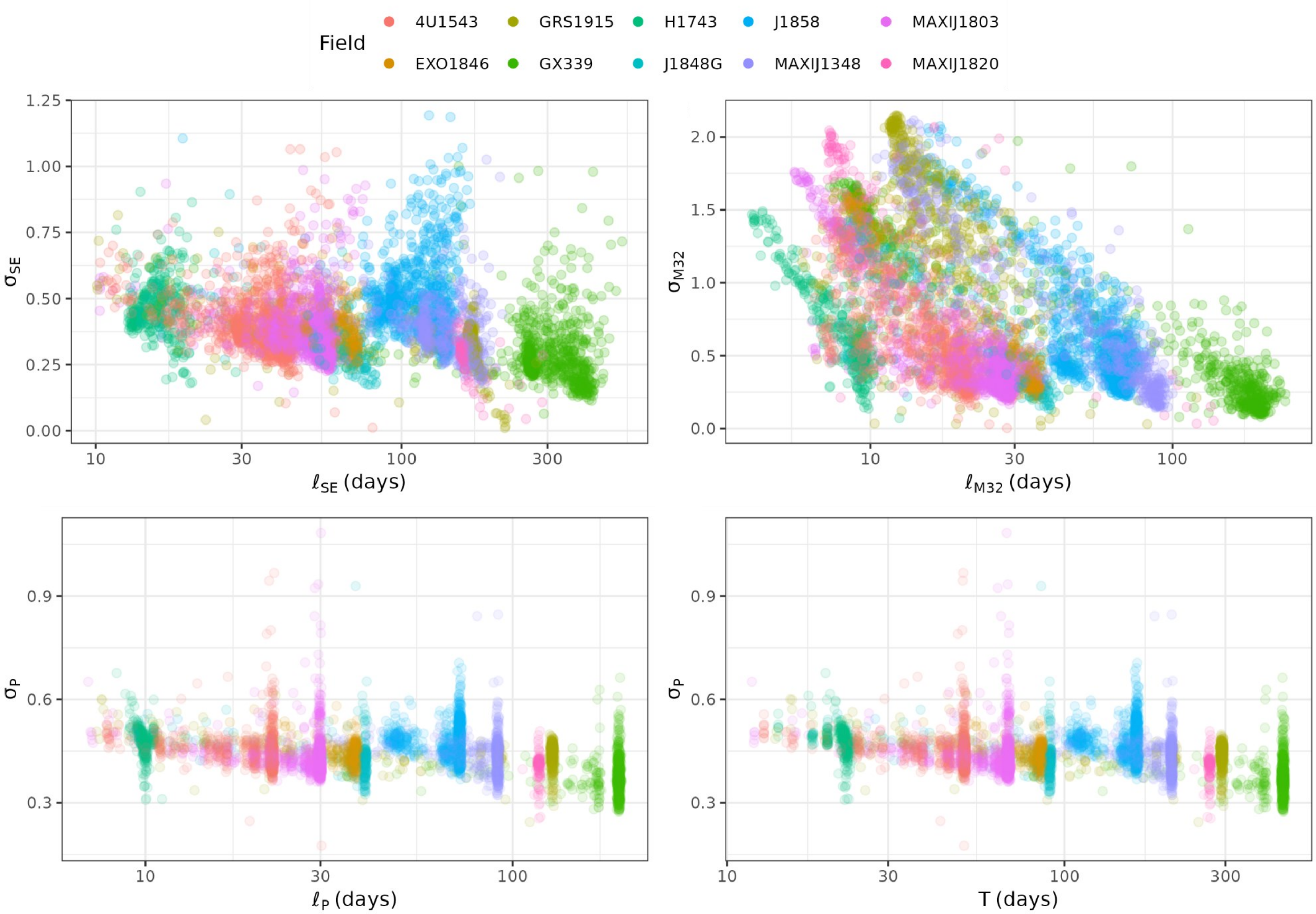
Posterior Predictive Curves



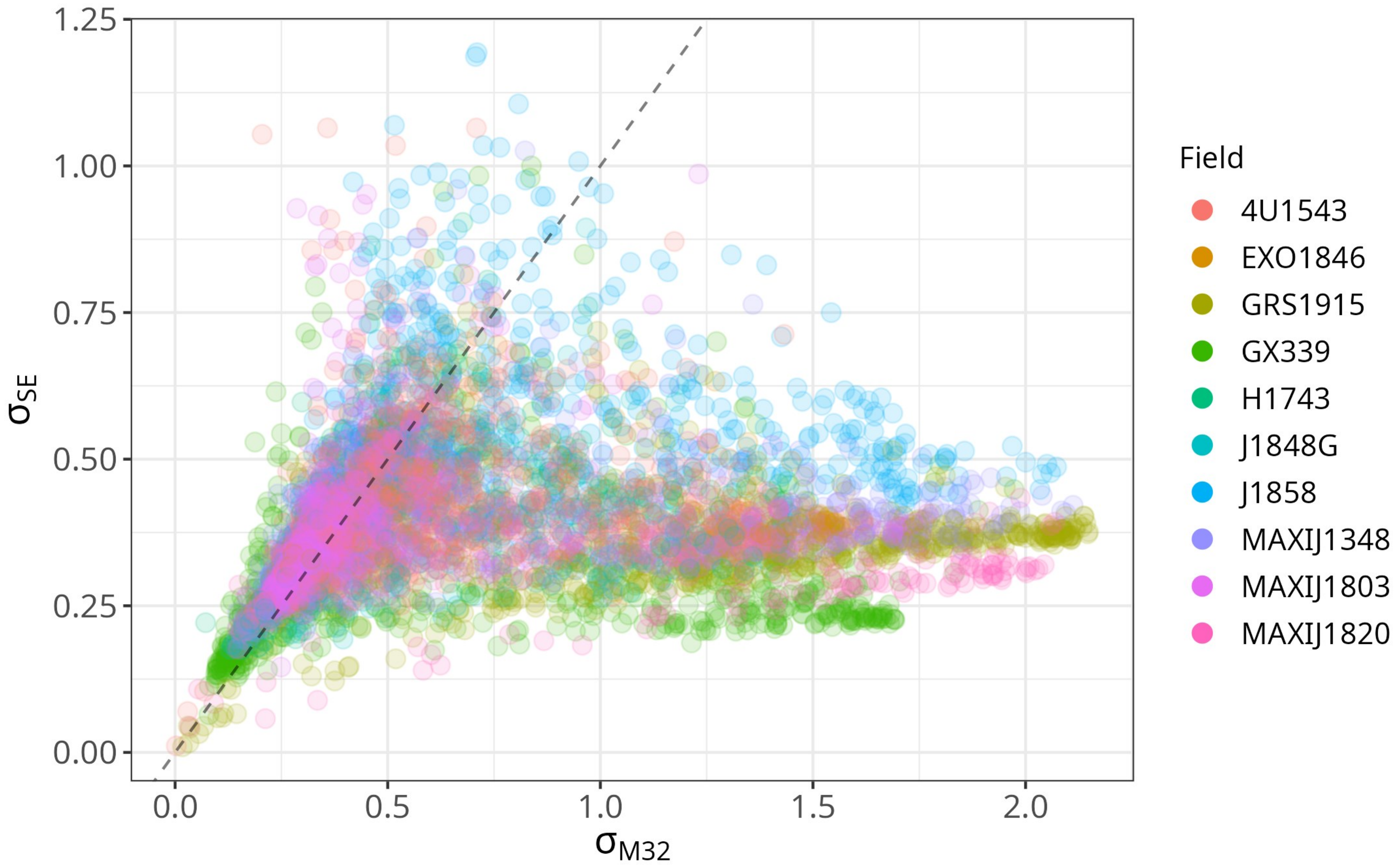
Posterior Medians

$\sigma_{SE} = 0.39$
$\sigma_{M32} = 1.26$
$\sigma_{P} = 0.50$
$\ell_{SE} = 50.0$
$\ell_{M32} = 11.9$
$\ell_{P} = 46.7$
$T = 41.1$

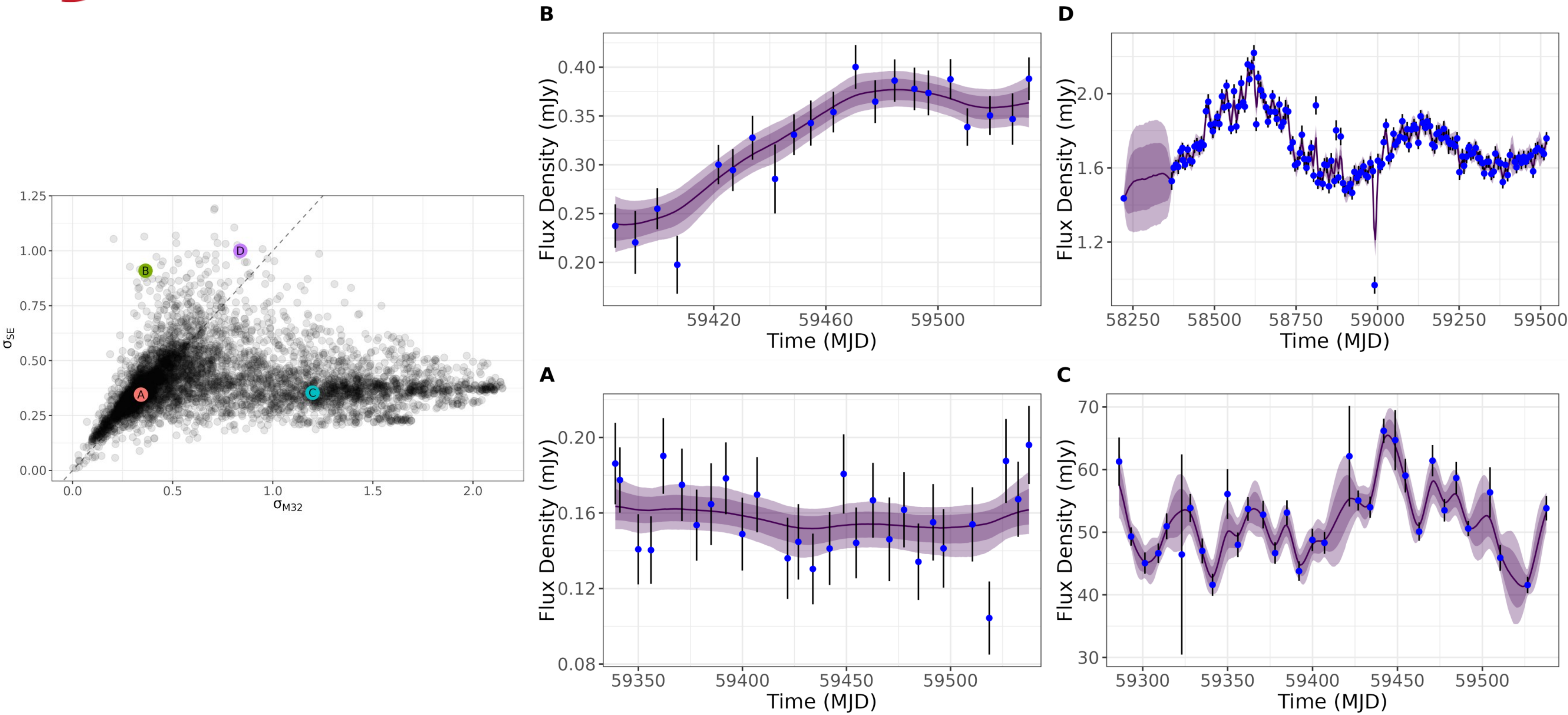
$\eta_{\nu} = 2.91$
$V_{\nu} = 0.12$


Field
4U1543
EXO1846
GRS1915
GX339
H1743
J1848G
J1858
MAXIJ1348
MAXIJ1803
MAXIJ1820
σ_{SE}
ℓ_{SE} (days)
σ_{M32}
ℓ_{M32} (days)
σ_P
ℓ_P (days)
σ_P
T (days)

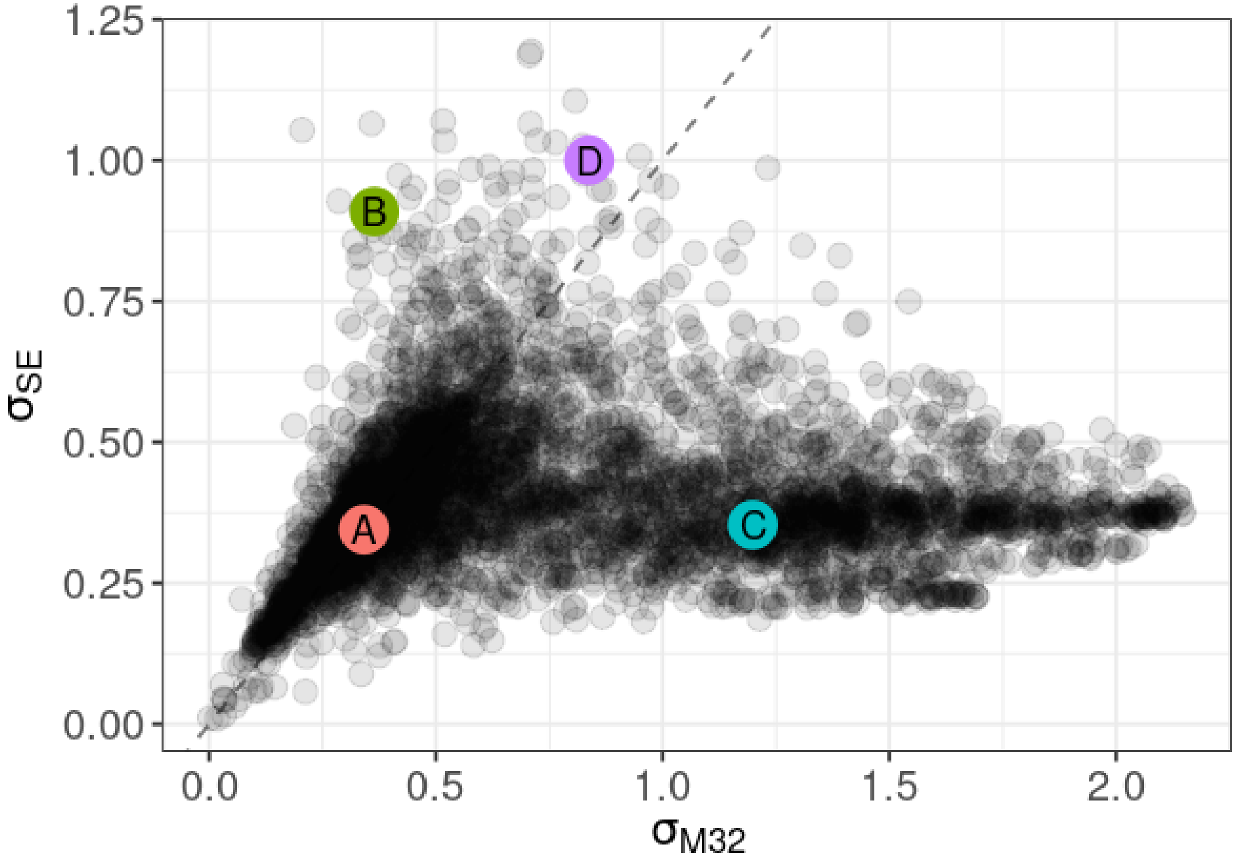
Amplitude Hyperparameter

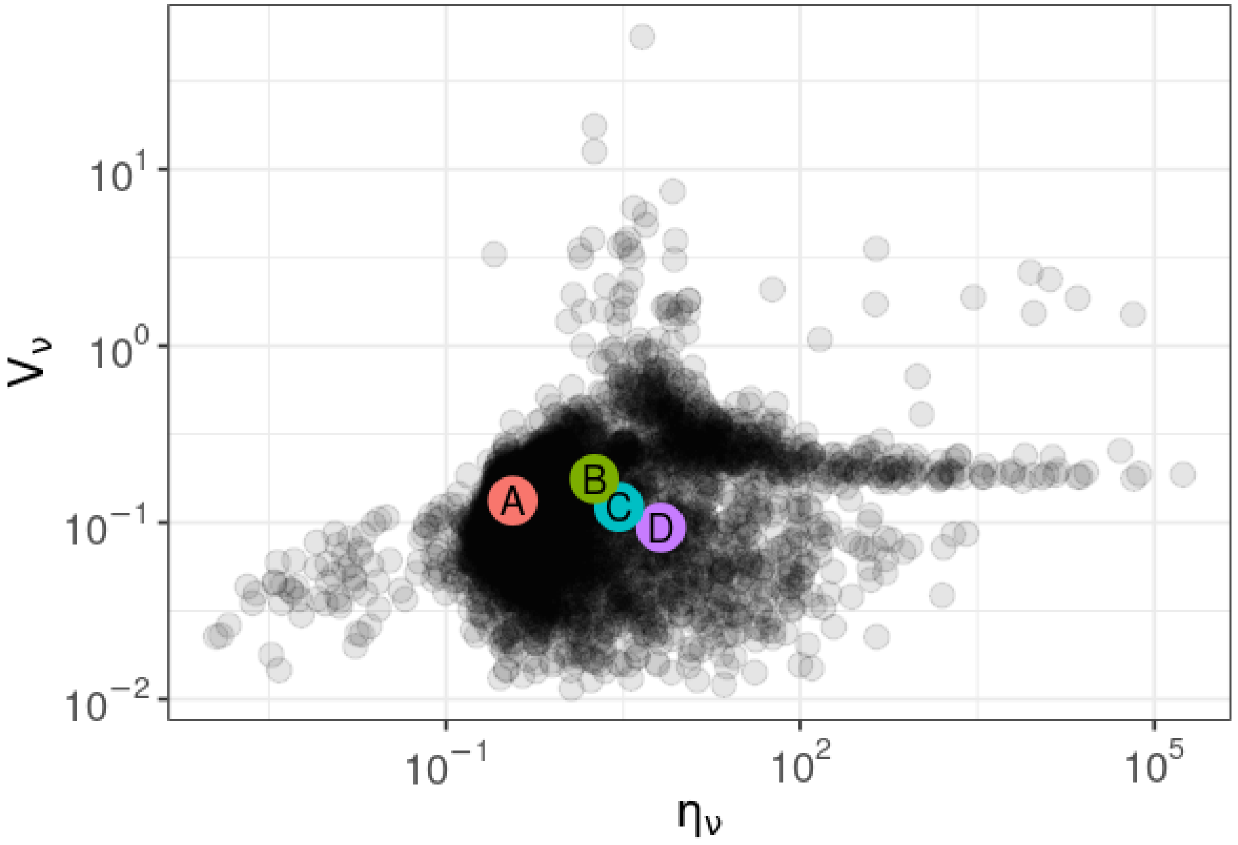


Explore the amplitude space



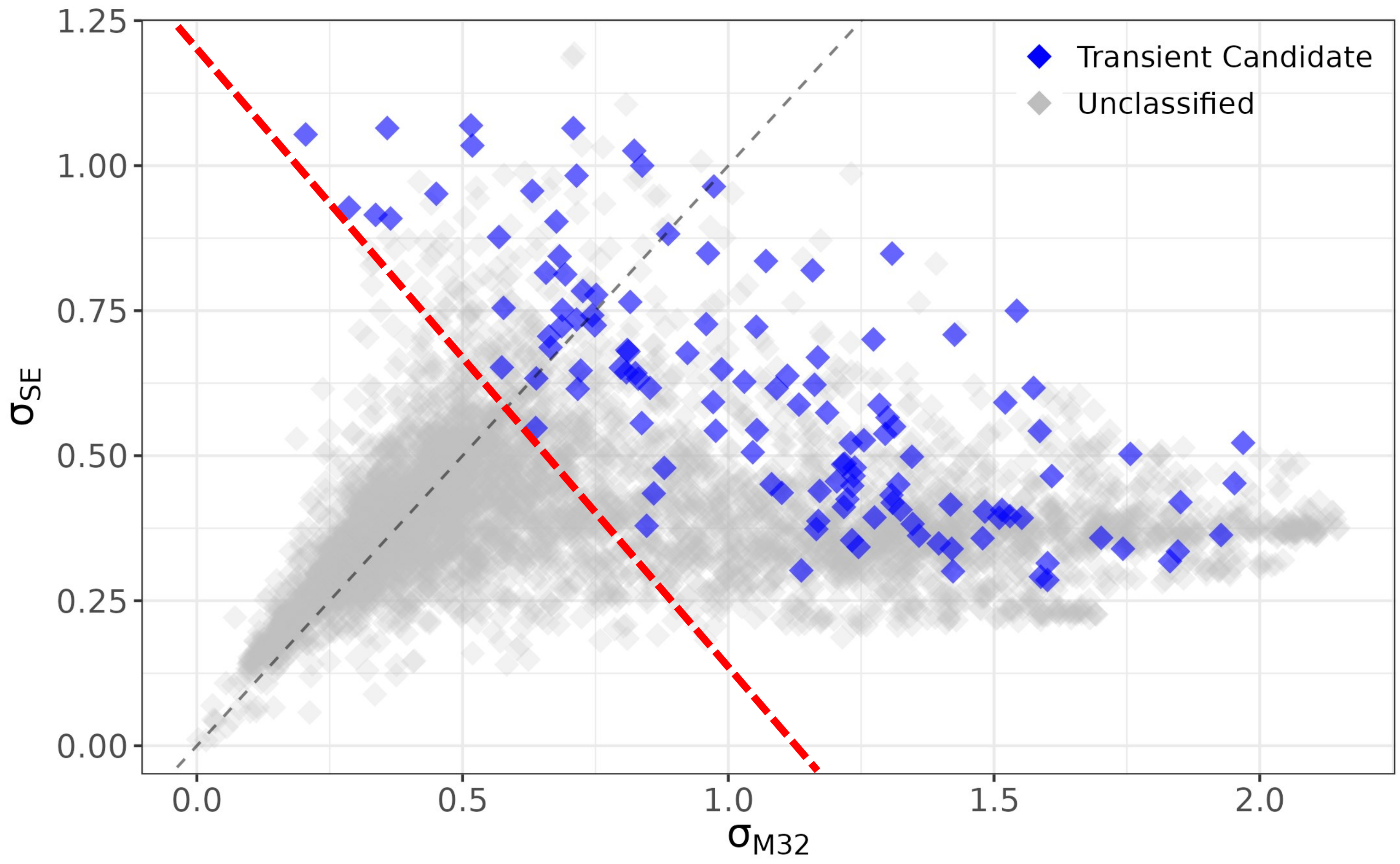
Comparison with (η_ν, V_ν)





Interpreting Amplitude as Transcience

- Transience seems to manifest as large values in **amplitude**, σ.
- Previously identified transient candidates all seem to lie the upper right of this parameter space.



Data: Andersson et al. (2023)

Figure: Fu et al. (in prep.)

Summary

- Developed models and code suitable for fitting univariate GPs to the light curves of a large radio survey, i.e., ThunderKAT.
- GPs can be used to perform hyperparameter inference as well as interpolation in time-domain astronomy.
- Amplitude hyperparameter, σ, is an interpretable descriptor of variability; more discriminatory than (η_ν, V_ν).
- Next: extend to multi-band light curves.

Twinkle twinkle little star… a Gaussian process is what you are!